Astrometry with fisheye lenses, using HRCAM as an example

Michael Ashley / UNSW / 11-March-2014

Our aim is to produce a transformation between RA and Dec and pixel coordinates on an all-sky image produced with a fisheye lens.

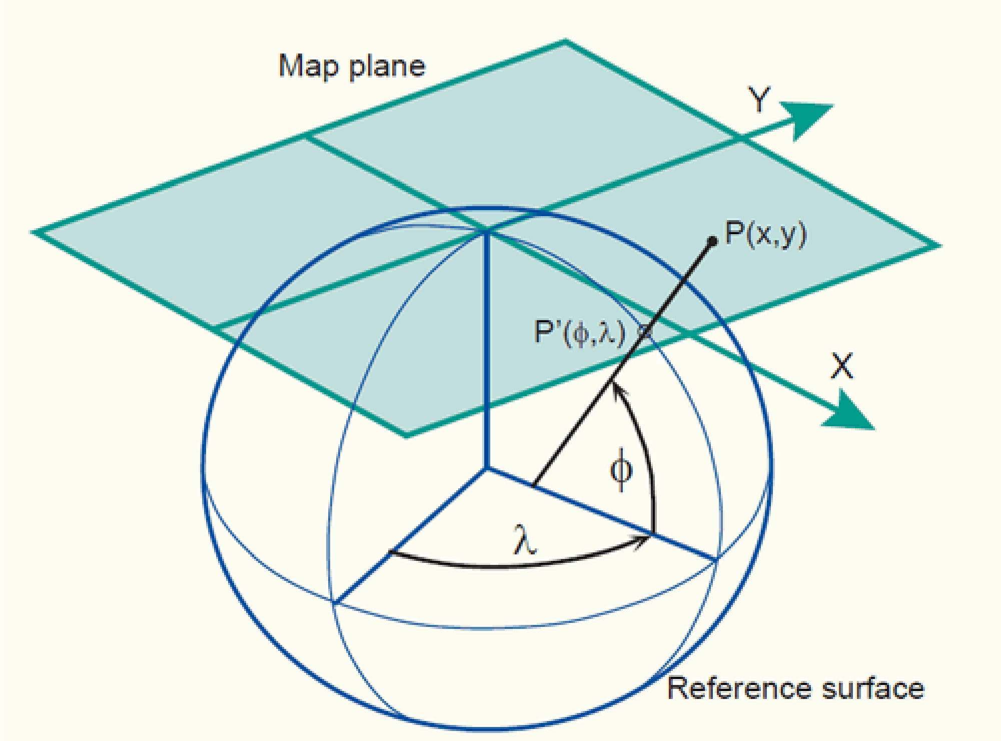

Figure 1: From http://kartoweb.itc.nl/geometrics/map %20projections/mappro.html

IRAF has routines for doing astrometry with “normal” astronomical images. By “normal” I mean images that cover a relatively small fraction of the sky, perhaps up to 10 degrees on a side. The approach used by IRAF begins with mapping (RA, Dec) on the sky to so-called “standard coordinates” (ζ,η) which are simply the coordinates on a plane that is tangent to the celestial sphere, touching at the centre of the image (see the Figure above). A reasonable number (typically more than 20 or so) of reference stars with known (RA, Dec) are identified in the image, and a least-squares fit is used to derive a transform between (ζ,η) and the coordinates (x,y) of the stars on the image. The fit can have as few as 6 parameters (2 offsets and 4 scales) if the transform is linear with rotation, or a dozen or so to take into account lens distortion. The parameters are known as “plate constants”.

With a sufficient number (hundreds) of reference stars, covering all parts of the image, it is usually the case that the residual fitting errors drop dramatically when the right choice is made of parameters to include in the fit.

The problem with all-sky images is that (ζ,η) goes to infinity for stars on the horizon, whereas on the image itself, there is clearly a limiting radius corresponding to the horizon.

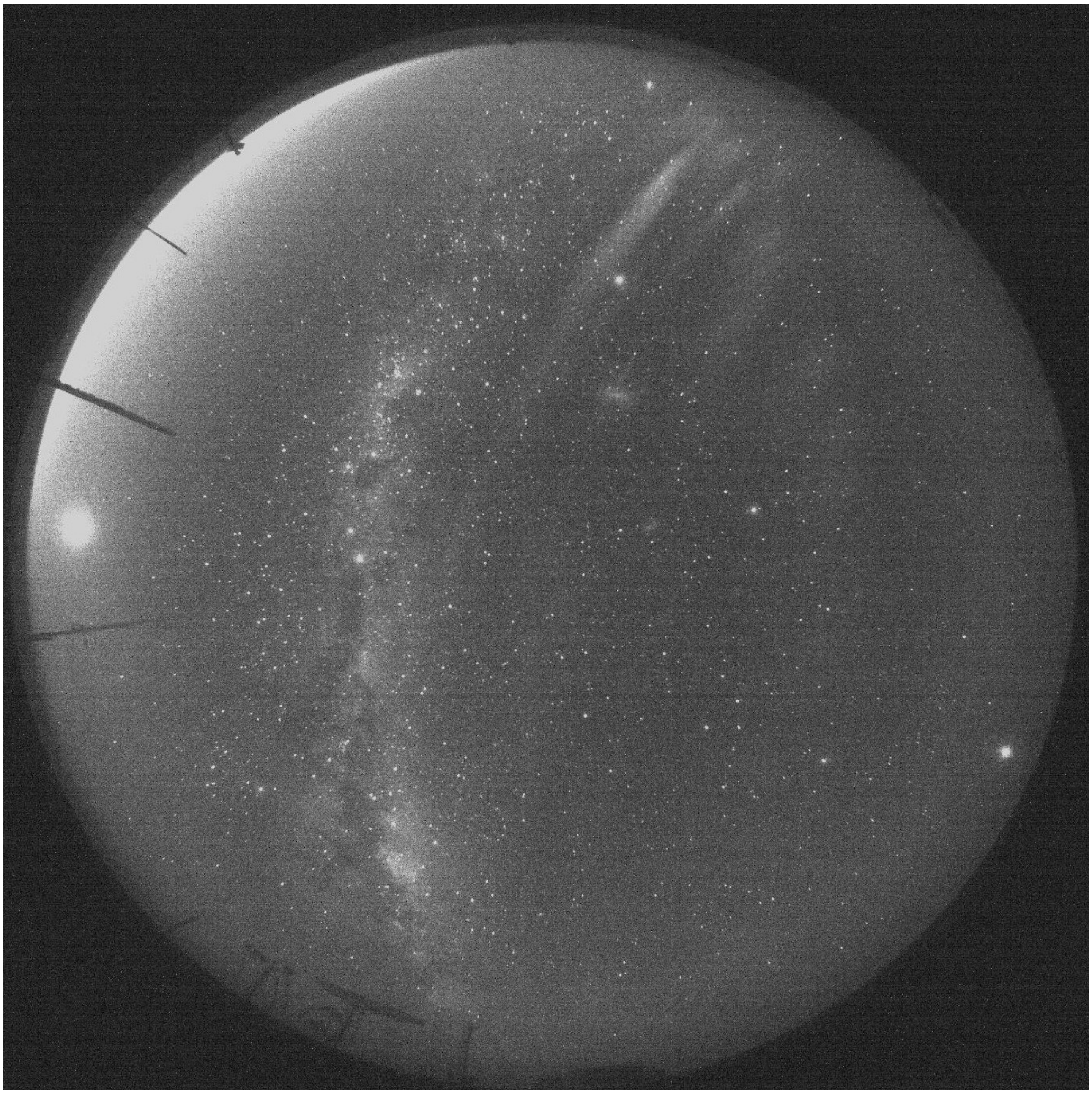

Figure 2: An example HRCAM image. This is extracted from a raw Canon image file, and is through one of the two G filters. The raw file contains a four images: one red, one blue, and two green. Each can be separately converted into FITS images and analysed.

The solution I have used with HRCAM is to map the (x,y) coordinates on the image to a coordinate system (let's call it (u,v)) which approximates (ζ,η), i.e., it is the transform of the (x,y) coordinates to a tangent plane to the celestial sphere. The mapping is empirically determined, and is expressed by a radial distortion function which sends points on the horizon to infinity. With this mapping, the usual IRAF tools can be used to determine the parameters relating (u,v) with (ζ,η) (and hence (RA, Dec)).

In the case of HRCAM, the radial function is

```
    r' = (1.0/450)*0.725e8*(-1.488e-3+(3.525e-6*r)+(-1.6e-9*r**2)+ 1/(665.7-r))
```

Where r is the radius from the centre of the image, and r' is the radius corrected for lens distortion. You can see that as r approaches 665.7 pixels, r' goes to infinity. This radius corresponds to the horizon. The radial function was determined empirically, by beginning with stars close to the centre of the image, fitting plate constants with IRAF, and then gradually working out towards the edge, adding more stars, and comparing the observed radial distance from the centre with that calculated using (ζ,η). Determining the radial function for a given lens is the trickiest part of the astrometry. Fortunately, it only has to be done once per lens, and may be similar for lens of the same model.

The astrometric data reduction process for HRCAM

The first step is to produce a catalog of stars that *might* fall in the image, i.e., the star coordinates are appropriate, and the stars magnitudes are within the observable range (we will later allow for the vignetting at the edges of the lens). To produce the star catalog we use the "tycho-ofek" program (written by mcba). This program has the Tycho catalog built into it, and also has a colour transform from Tycho magnitudes to Canon filters. Here are its usage instructions:

```
Usage: ./tycho-ofek ra_centre dec_centre delta_ra delta_dec magID min_mag
                    max_mag magID...
        all angles in radians. magID is b,v,bt,vt,j,h,k,g,r,i,z,g2,r2,i2,or z2
        The output format is: ra,dec,mags...,starNumber
```

For example, the following line produces a list of the 2096 Tycho stars between VT magnitudes -2 and 5.9 that are likely to be in an HRCAM image.

```
   ./tycho-ofek 5.200 -1.441 3.2 1.5 vt -2 5.9 b v bt vt r
```

“`b v bt vt r`” are the magnitudes that are output.

You can download the program here: tycho-ofek.c and you will also need proc-ofek.pl which is a perl script to convert Ofek's revised Tycho-2 catalog into a C header file.

The centre of the HRCAM images is at about RA = 5.20 rads, Dec = -1.44 rads. Here is a small sample of the output from tycho-ofek, with column headings added:

```
# RA         Dec           B      V      BT     VT     r    Star no.
5.53610101 -1.55258379   5.74  5.43  5.77  5.47  5.55      78
4.95143562 -1.52901041   6.58  5.30  6.98  5.43  4.95     545
6.14383428 -1.52685266   6.78  5.51  7.18  5.64  5.04     586
5.89757743 -1.50041159   6.78  5.77  7.07  5.87  5.55    1596
```

Next, we take the output from “tycho-ofek” and pass it to ./proc-tycho.pl, which calculates the standard (ζ,η) coordinates of the stars through knowledge of the observation epoch of the image. (ζ,η) coordinates depend only on the (RA, Dec) of the centre of the lens and the local sidereal time (LST, calculated from the UNIX epoch of the image in seconds).

```
   ./tycho-ofek 5.200 -1.441 3.2 1.5 vt -2 5.9 b v bt vt r |
            ./proc-tycho.pl -epoch 1283530500
```

Here is a small sample of the output from the previous command, with headings added:

```
#      xi           eta     RA          Dec         B      V    BT   VT     r    Star No.
     0.412       -7.485 5.53610101 -1.55258379 5.74 5.43 5.77 5.47 5.55 78
    -0.436       -6.071 4.95143562 -1.52901041 6.58 5.30 6.98 5.43 4.95 545
     2.150       -7.100 6.14383428 -1.52685266 6.78 5.51 7.18 5.64 5.04 586
     2.804       -5.506 5.89757743 -1.50041159 6.78 5.77 7.07 5.87 5.55 1596
```

Now, what we really want are the actual (x,y) positions of the stars on the image (as opposed to the standard coordinates (ζ,η)), so we run ./proc-tycho.pl again with the “-inv” switch. The resulting coordinates are corrected for the fisheye lens radial distortion.

./proc-tycho.pl -inv also applies a magnitude cut-off with allowance for the known vignetting of the lens, so that fainter stars at the edges are not included. -magpos is the column containing the magnitude, where 1 is the column with the first magnitude ('b' in this case).

The above sequence can be compressed into one command line, with the output of each program piped into the input of the next:

```
./tycho-ofek 5.200 -1.441 3.2 1.5 vt -2 5.9 b v bt vt r |
            ./proc-tycho.pl -epoch 1283530500 |
            ./proc-tycho.pl -inv -magpos 4 -maglim 9 > ref
```

```
#   x         y       xi      eta    RA          Dec         B     V    BT    VT    r    Star No.
1190.829  748.012  0.412 -7.485 5.53610101 -1.55258379 5.74 5.43 5.77 5.47 5.55 78
1183.992  759.656 -0.436 -6.071 4.95143562 -1.52901041 6.58 5.30 6.98 5.43 4.95 545
1204.872  751.132  2.150 -7.100 6.14383428 -1.52685266 6.78 5.51 7.18 5.64 5.04 586
1210.069  764.183  2.804 -5.506 5.89757743 -1.50041159 6.78 5.77 7.07 5.87 5.55 1596
```

Note that stars that fall off the image are not included in the output.

The (*x,y*) coordinates should now be directly comparable to positions on the HRCAM images, and you can overlay the coordinates of the reference stars on the image with IRAF as follows:

```
display
h1283530500-G1 1
tvmark 1 ref
mark=circle
radii=5
```

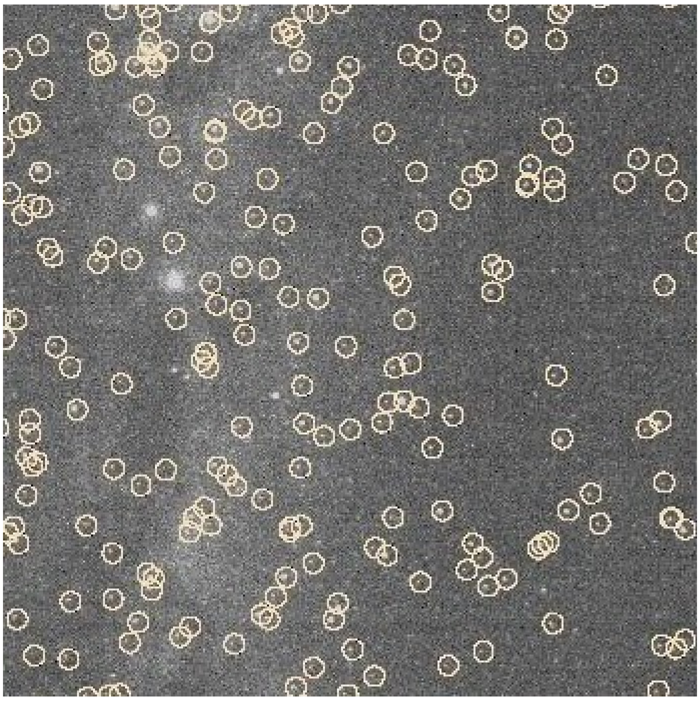

Figure 3: A small section of the previous HRCAM image showing the identified reference stars. The circles are positioned purely on the basis of the image timestamp, the known image centre, and the empirical radial distortion of the lens. The two bright stars in the 2nd quadrant are alpha and beta centauri.

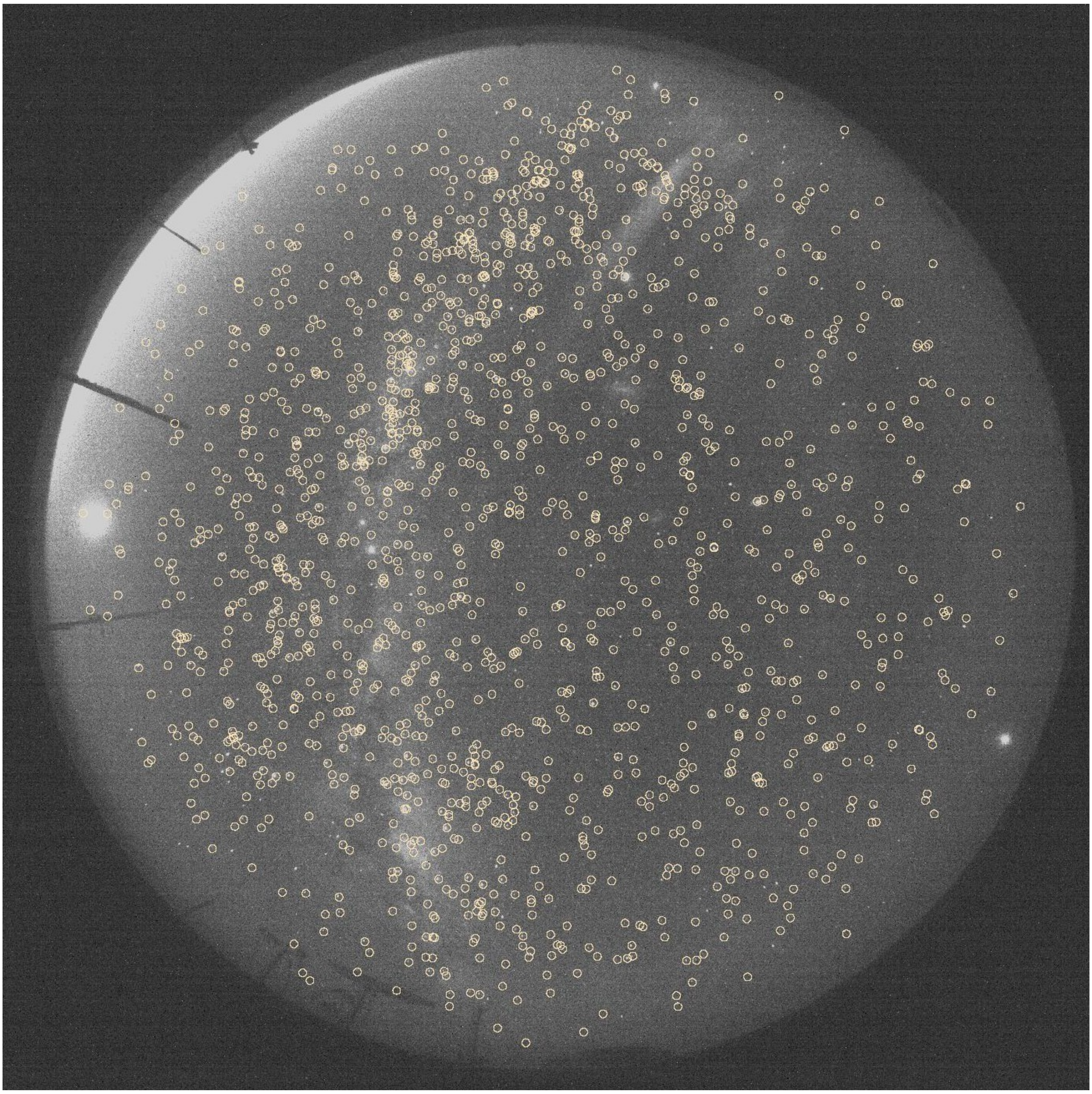

Figure 4: The full HRCAM image showing the positions ofthe Tycho reference stars. The bright object at the right, below the centre, is Jupiter.

The next step is to use IRAF to produce a catalog of the star-like objects that are actually found in the image. This can be done with

```
starfind image=h1283530500-G1 output=stars.0 hwhmpsf=0.80 thresh=60 frad=2.5
              sepmin=6 npix=5 roundlo=0.0 roundhi=0.6 sharplo=0.7 sharphi=1.4
```

We then remove the preamble of comments that IRAF produces, and filter the output to only include objects within the fisheye lens field-of-view (this is so that we don't inadvertantly use noise spikes / cosmic ray artifacts that are outside the FOV):

```
cat stars.0 | ./proc-stars.pl > stars
```

The file “stars” looks as follows (I've added a header for the purposes of this document). ID' is the star number from IRAF “starfind”, ID is the number in the file “stars”.

```
#   X          Y          Mag      Area  FWHM  Round    Pa     Sharp    ID' ID
1113.912   145.216     -5.26       5   0.68  0.272   92.0    0.855    604  1
1269.194   144.912     -5.89      12   0.88  0.070   80.5    1.096    605  2
1286.788   145.124     -5.62       9   0.88  0.206  171.8    1.103    606  3
1145.898   146.138     -5.54       7   0.73  0.599   74.8    0.915    616  4
```

At this point we have a list of stars found by IRAF, and a list of stars predicted using the Tycho catalog. We now have to combine these two lists into one, matching the stars. This somewhat non-trivial job is handled by the rather cunning program “match.c” (written by mcba). Here is some sample output:

```
# x          y      rad     rad'      RA           Dec        zd airmass   mag  B    V    VT   BT    r       xref     yref    fit nc r#   s# n#
1222.819 278.020  531.43  139.59 1.89957784 -0.53557947 1.201 2.748 -5.18 5.25 5.31 5.14 5.31 5.59 1235.833 287.342 1.42 1 185 1175 1
1203.920 290.898  517.76  129.84 1.93527159 -0.56203217 1.173 2.569 -5.94 5.30 5.37 5.18 5.37 5.68 1217.858 296.223 3.17 2 212 1141 2
1285.240 292.846  524.19  135.86 1.76485268 -0.54228293 1.196 2.715 -5.29 5.18 5.20 5.07 5.21 5.30 1299.689 300.610 1.96 3 216 1166 3
```

```
 where:
      x      is the x coordinate in the image, from "stars"
      y      is the y coordinate in the image, from "stars"
      rad    is the radius in pixels of the star from the centre of the lens
      rad'   is the radius from the centre of the star using (xi,eta) coordinates
      RA     is the right ascension of the reference star, from "ref"
      Dec    is the declination of the reference star, from "ref"
      zd     is the zenith distance of the star (corrected for refraction)
      airmass is the airmass of the star (using the slalib algorithm)
      mag    is the measured star magnitude, from "stars"
      B      is the B mag, from "ref"          V      is the V mag, from "ref"
      BT     is the Tycho B mag, from "ref"
      VT     is the Tycho B mag, from "ref"
      r      is the r mag, from "ref"
      xref   is the x coordinate of the reference star, from "ref"
      yref   is the y coordinate of the reference star, from "ref"
      fit    is the average absolute value of the "actual"-"estimated"
                x and y distances from each of the closest
                neighbouring reference stars,
                the smaller this is, the more likely the star ID is correct
      nc     is the number of closest neighbours used
      r#     is the ID in "ref"
      s#     is the ID in "stars"
      n#     is the ID in this file (i.e., the line number)
```

```
/home/iraf/apt/bin.linux64/tycho 5.200 -1.441 3.2 1.5 -2 6.3 |
       ./proc-tycho.pl -epoch 1279044240 |
       ./proc-tycho.pl -inv >ref && ./a.out >k.2

h1283530500-G1:

time /home/iraf/apt/bin.linux64/tycho 5.200 -1.441 3.2 1.5 -2 5.3 |
       ./proc-tycho.pl -epoch 1283530500 |
       ./proc-tycho.pl -inv >ref && ./a.out > id.txt
```

Finally, we can use IRAF aperture photometry to find star magnitudes, the sky, etc. This is done with the IRAF task “HRPhot.cl”:

```
apt
task HRPhot = apt$HRPhot.cl
HRPhot h1283530500-G1
```

HRPhot.cl uses the star positions from “id.txt”, and outputs photometry in “phot.txt”, with summary data in “tmp.txt”.

```
# Xcen     Ycen   ID   Xshift Yshift   Sky       Mag1    Mag2    Mag3
1374.705  222.037  1  -0.113   0.198  1068.997  19.927  19.761  19.639
1402.365  222.420  2  -1.616  -1.748  1065.63   20.533  20.263  19.942
1114.434  241.243  5  -0.511   0.909  1086.156  20.734  20.424  20.270

        Xcen   is the X coordinate of the centre of the photometry aperture
        Ycen   is the Y coordinate of the centre of the photometry aperture
        ID     is the star ID from "stars"
        Xshift is the shift in the X coordinate from the value in id.txt
        Yshift is the shift in the Y coordinate from the value in id.txt
        Sky    is the sky background around the star in ADU per pixel
        Mag1   is the magnitude in an aperture of 1.2 pixels
        Mag2   is the magnitude in an aperture of 1.5 pixels
        Mag3   is the magnitude in an aperture of 1.8 pixels
```

We then process “tmp.txt” to add additional columns from “id.txt” and produce a final file “out.txt” which is just “tmp.txt” with the info from “id.txt” appended to the end of each row:

```
./proc-out.pl > out.txt

#  Xcen      Ycen    ID   Xshift Yshift   Sky       Mag1    Mag2    Mag3    x
y       rad     rad'     RA          Dec       zd airmass  mag  B    V   VT   BT
r      xref    yref    fit nc  r#  s#  n#
1361.738  1335.325  552  -0.017  0.035  1055.138  18.819  18.584  18.436
1361.755  1335.290  553.77  181.89 5.44401299 -0.16573256 1.239 3.046 -6.61 3.83
3.74 3.77 3.76 3.97 1366.431 1335.296 0.62 9 1033 964 552
```

The entire process is automated for one image (of the four within a raw Canon image file) by HR.cl, which processes one colour image and produces an “out.txt” file. This program also determines the dark (Dk) mean, median and standard deviation, and the sky mean, median and standard deviation in four regions (S0, S1, S2, S3) in the bright band around the horizon. The values are appended to the file “res”, along with summary statistical information from running “proc-id.pl” on “id.txt”:

```
#Epoch Colour Ap CT ISO Exp Dk mean Dk median Dk stddev S0mean  S0median
S0stddev  S1mean  S1median S1stddev S2mean S2median S2stddev S3mean  S3median
S3stddev:
1270659969-G1 2.8 6 400 120 1025.008 1024.733 8.095572 1514.277 1514.443
30.73497 1691.389 1692.37 35.7622 1511.426 1511.652 28.41977 2208.05 2203.661
85.43636:
 797 stars; delta mag = 4.26 (631 stars, raw = 4.17); shift = (-2.13, -1.77);
rotation = 0.009
```

For h1279044240-R.fits:

The entire process is automated for all four colours of a list of images by HRList.cl. Which takes as input a list of images, e.g.,

```
HRList @a
```

where “a” is a file with one image per line, e.g.,

```
/data1/hrcam/h1270051440
/data1/hrcam/h1270055040
/data1/hrcam/h1270058640
```

“a” can be produced using the program ./proc-hrcam-list.pl