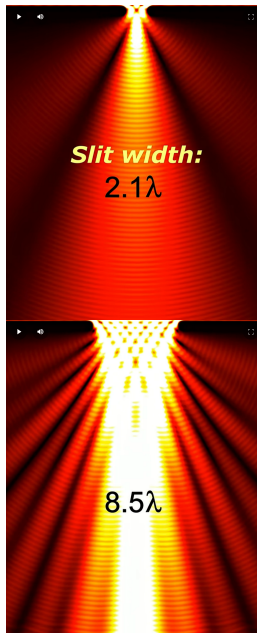
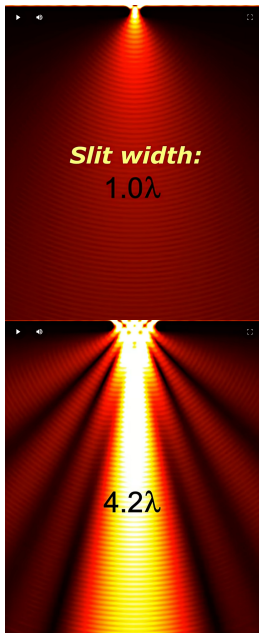


# Lecture 13—learning outcomes

- Sketch a pattern showing the intensity of light passing through a double slit. Recognise which features in the pattern are due to the diffraction from the slits and which parts are due to the interference of the light passing through the two slits.
- Describe and sketch the diffraction patterns from small circular apertures or obstacles.
- Apply Rayleigh's criterion to determine whether the light from distant sources is resolvable.
- Explain how a diffraction grating works. Calculate the locations of the maxima in the diffraction pattern.
- Give a definition of dispersion and resolving power and explain how these concepts relate to diffraction gratings. Calculate these given appropriate quantities.
- Explain how x-ray diffraction can be used to calculate the interplanar spacing inside a crystal. For a given crystal describe the diffraction pattern.

Textbook chapters: 36.3–36.7 <https://ebookcentral.proquest.com/lib/unsw/reader.action?c=UERG&docID=30765671&ppg=888>

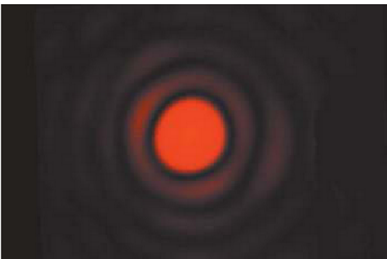
# Visualising single slit interference



The video <https://www.youtube.com/shorts/eSds5I62nu0> at left shows laser light passing through a single slit, with various ratios of slit width to  $\lambda$ . Note the following:

- With  $a = \lambda$ , there is a broad maximum in the straight-ahead direction, and darkness at  $\pm 90^\circ$ .
- Fringes become more obvious when  $a \gtrsim 2\lambda$ .
- The number of dark fringes is  $2a\lambda$ . You can derive this from the single slit equation for minima:  
 $a \sin \theta = m\lambda$  where  $m = 1, 2, 3, \dots$
- Note the complexity of the interference in the Fresnel region close to the slit.
- Note the relative simplicity of the interference in the Fraunhofer region far from the slit.

# Diffraction by a circular aperture



- The top figure shows Fraunhofer (far-field) diffraction from a laser beam hitting a small aperture in an opaque screen. It can be shown (with some tricky mathematics) that the angle  $\theta$  from the central peak to the first minimum in the diffraction pattern is

$$\sin \theta = 1.22 \frac{\lambda}{a} \quad (\text{circular aperture})$$

where  $d$  is the diameter of the aperture.

- Compare this with the formula for a slit of width  $a$

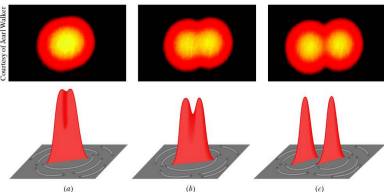
$$\sin \theta = \frac{\lambda}{a} \quad (\text{slit})$$

- The lower figure shows the application of this to the resolution of an optical system, e.g., observing two stars with a telescope. Given that  $\theta$  is small, we define Rayleigh's criterion for the resolvability of two stars as

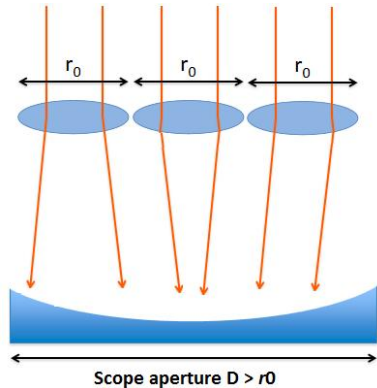
$$\theta_R = 1.22 \frac{\lambda}{d} \quad (\text{Rayleigh's criterion})$$

This is also known as the **diffraction limit** of the telescope.

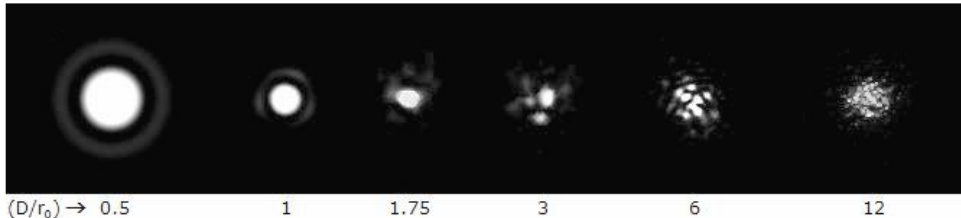
- When two stars are separated by Rayleigh's criterion the peak of one coincides with the minimum of the other.



# The resolution of ground-based astronomical telescopes



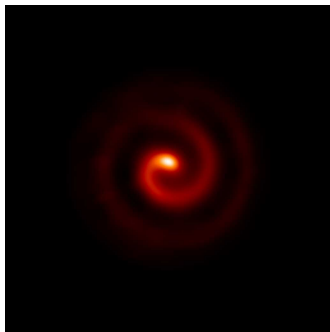
- Given that the resolution of a telescope of diameter  $D$  scales as  $1/D$ , you would think that, e.g., a 10 m diameter telescope would be able to resolve features 100 times smaller than a 0.1 m telescope. Unfortunately, the Earth's atmosphere gets in the way.
- The figure on the left shows a simplification of the atmosphere above a telescope mirror where the three ovals of diameter  $r_0$  represent areas of different density (and hence different refractive index).
- $r_0$  is known as the Fried parameter, within which the phase shifts from the atmosphere are less than 1 radian. Telescopes with diameters less than  $r_0$  can achieve the diffraction limited resolution. Once the diameter exceeds  $r_0$  there is no improvement, since the wavefront is no longer coherent.
- At a good astronomical site,  $r_0$  is typically about 200 mm.
- The figure on the bottom shows a series of 1 ms exposures of star images as a function of  $D/r_0$ . The individual dots are known as **speckles**. Exposures of  $> 1$  s would blur the speckles together, essentially producing the image at  $D/r_0 = 1$ .



# Breaking the atmospheric limit



Peter Tuthill with an aperture mask for the 10 m Keck telescope

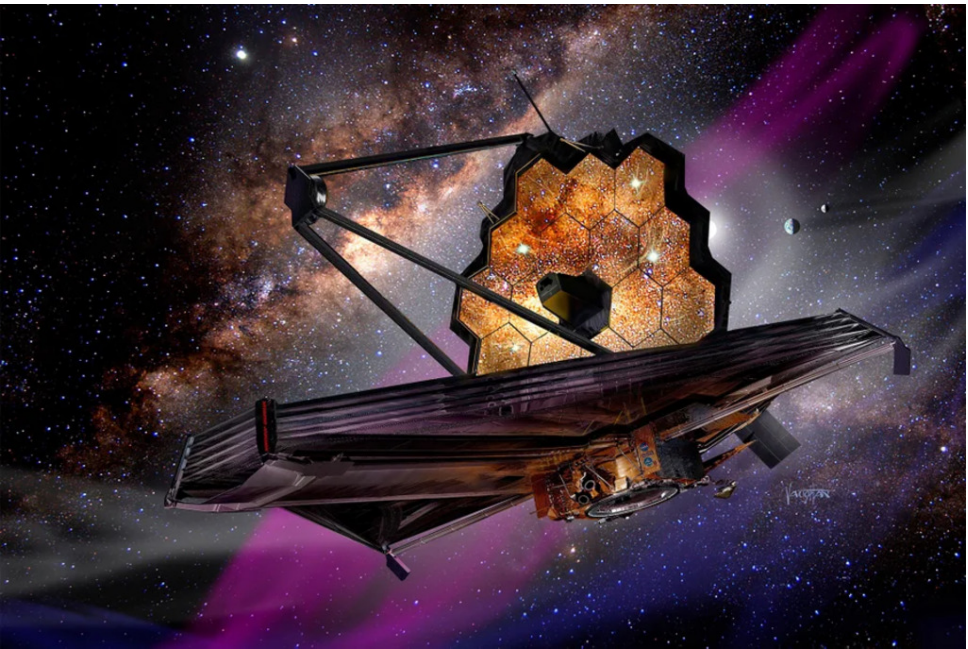


[https://www.physics.usyd.edu.au/~gekko/pinwheel/movie\\_11.gif](https://www.physics.usyd.edu.au/~gekko/pinwheel/movie_11.gif)

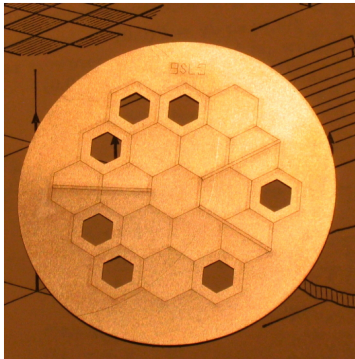
gif

- Astronomers have come up with a number of techniques to circumvent the atmospheric limitation to resolution.
- The image at left shows Australian astronomer Peter Tuthill from USyd inside the 10 m Keck telescope, holding a sparse aperture mask. With some clever software this enables high resolution images like the one shown at bottom left to be produced. This image exceeds the atmospheric limit by more than a factor of  $\sim 50$ .
- Another technique is called **adaptive optics**, where the telescope contains a deformable optical element able to change shape at  $\sim 100$  Hz speeds to take out the atmospheric wavefront errors.
- Both of these techniques have limitations, but they do enable resolutions almost equal to the diffractional limit in certain circumstances.
- A better option is to put the telescope in Antarctica, where the stable atmosphere results in  $r_0$  being better by factors of 2–4.

# Aperture masks can even help in space



# The JWST NIRISS instrument with AMI mask

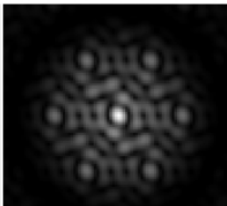


- The prototype aperture mask at left can be used with the JWST NIRISS instrument to improve the resolution beyond the Rayleigh limit.
- The Rayleigh limit uses the full aperture, and so averages contributions across it. The mask puts emphasis on the most widely separated parts of the aperture, and achieves  $\sim 2.5$  times increase in resolution.
- The simulated diffraction patterns from a star observed by the JWST are shown in the bottom image.

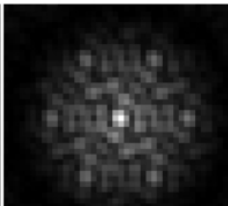
Full-Aperture PSF  
(Theoretical)



Masked PSF  
(Theoretical)



Masked PSF  
(Detector  
Resolution)



# The resolution of the human eye



Maximilien Luce, *The Seine at Herblay*, 1890. Musée d'Orsay, Paris, France. Photo by Erich Lessing/Art Resource

## A pointillistic painting by Maximilien Luce

The painting above is made from individual dots of colour, which are only visible as separate dots if you are close enough to resolve them.

The diameter of the eye's pupil sets the maximum resolution for the human eye. In bright light the pupil's diameter can be as small as 2 mm, and in the dark it can increase to 8 mm.

While your theoretical resolution will therefore be better in the dark, it can be reduced by aberrations (departures from perfect optics) in the eye that become more obvious with the wider pupil.

# Problem—resolution of the human eye

Assume that the average center-to-center separation of the dots in a pointilistic painting is  $D = 2.0 \text{ mm}$ . Also assume that the diameter of the pupil of your eye is  $d = 1.5 \text{ mm}$  and that the least angular separation between dots you can resolve is set by Rayleigh's criterion. What is the *least viewing distance* from which you cannot distinguish any dots in the painting?

# Solution

## Sample Problem 36.3.1 Pointillistic paintings use the diffraction of your eye

Figure 36.3.4a is a representation of the colored dots on a pointillistic painting. Assume that the average center-to-center separation of the dots is  $D = 2.0$  mm. Also assume that the diameter of the pupil of your eye is  $d = 1.5$  mm and that the least angular separation between dots you can resolve is set only by Rayleigh's criterion. What is the least viewing distance from which you cannot distinguish any dots on the painting?

FCP

### KEY IDEA

Consider any two adjacent dots that you can distinguish when you are close to the painting. As you move away, you continue to distinguish the dots until their angular

separation  $\theta$  (in your view) has decreased to the angle given by Rayleigh's criterion:

$$\theta_R = 1.22 \frac{\lambda}{d}. \quad (36.3.4)$$

**Calculations:** Figure 36.3.4b shows, from the side, the angular separation  $\theta$  of the dots, their center-to-center separation  $D$ , and your distance  $L$  from them. Because  $D/L$  is small, angle  $\theta$  is also small and we can make the approximation

$$\theta = \frac{D}{L}. \quad (36.3.5)$$

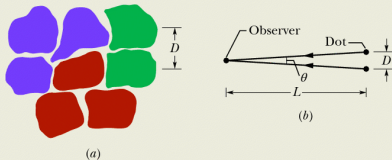
Setting  $\theta$  of Eq. 36.3.5 equal to  $\theta_R$  of Eq. 36.3.4 and solving for  $L$ , we then have

$$L = \frac{Dd}{1.22\lambda}. \quad (36.3.6)$$

Equation 36.3.6 tells us that  $L$  is larger for smaller  $\lambda$ . Thus, as you move away from the painting, adjacent red dots (long wavelengths) become indistinguishable before adjacent blue dots do. To find the least distance  $L$  at which *no* colored dots are distinguishable, we substitute  $\lambda = 400$  nm (blue or violet light) into Eq. 36.3.6:

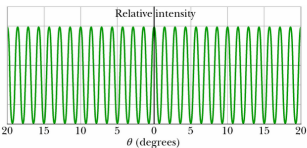
$$L = \frac{(2.0 \times 10^{-3} \text{ m})(1.5 \times 10^{-3} \text{ m})}{(1.22)(400 \times 10^{-9} \text{ m})} = 6.1 \text{ m.} \quad (\text{Answer})$$

At this or a greater distance, the color you perceive at any given spot on the painting is a blended color that may not actually exist there.

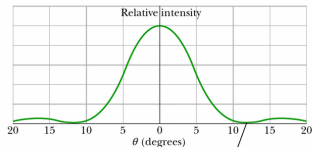


**Figure 36.3.4** (a) Representation of some dots on a pointillistic painting, showing an average center-to-center separation  $D$ . (b) The arrangement of separation  $D$  between two dots, their angular separation  $\theta$ , and the viewing distance  $L$ .

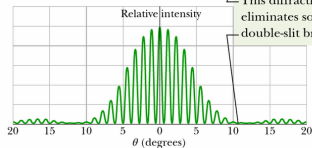
# Diffraction by a double slit



(a)



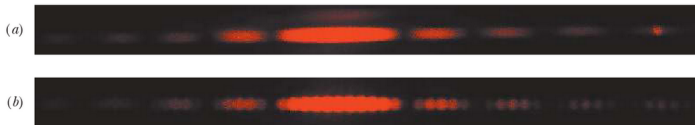
(b)



(c)

- In our previous discussions of the double-slit experiment, we implicitly assumed that the slits were much narrower than the wavelength of the light illuminating them, i.e.,  $a \ll \lambda$ . This results in an intensity pattern like the top panel in the figure, where the fringes are constant in amplitude.
- If the slit widths are not much less than  $\lambda$  you have to take into account single-slit diffraction, an example of which is shown in the middle panel.
- The net effect is to modulate the double-slit interference fringes by the single-slit diffraction, as shown in the bottom panel.
- Note that the first minima of the single-slit diffraction pattern eliminates the double-slit fringes that would have occurred near  $12^\circ$ .

# Diffraction by a double slit



Courtesy Jearl Walker

- The figure shows photos of the screen from a double-slit experiment illuminated by a laser.
- The top panel shows what you see if one of the two slits is covered. This is single slit diffraction.
- The bottom panel shows both slits uncovered. Now you can see the more closely spaced fringes resulting from the double slit interference.
- The intensity of a double-slit pattern where  $a$  is the slit width, and  $d$  is the distance between the slit centres is

$$I(\theta) = I_m (\cos^2 \beta) \left( \frac{\sin \alpha}{\alpha} \right)^2$$

where

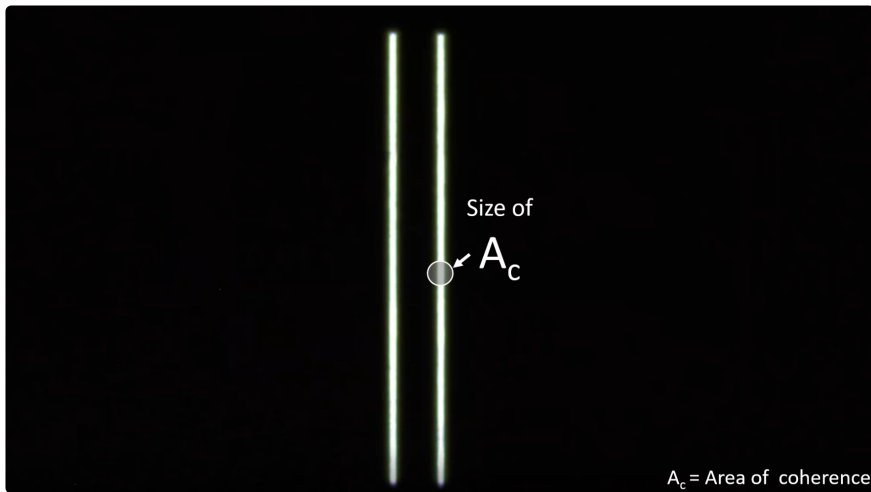
$$\beta = \frac{\pi d}{\lambda} \sin \theta$$

and

$$\alpha = \frac{\pi a}{\lambda} \sin \theta$$

# Double-slit experiment—the importance of spatial coherence

The video linked below shows some interesting observations of the double-slit experiment when the coherence length is less than or greater than the slit separation. Also, it calculates the coherence length of sunlight at the distance of the Earth.



Light & Coherence part 2: Spatial Coherence (and the Double Slit Experiment)



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Clip



<https://www.youtube.com/watch?v=nba4ztLBEh0>



# The distinction between diffraction and interference

The term **interference** is used when we have a small number of coherent sources, such as in the double slit experiment.

The term **diffraction** is used when the combining waves originate in a single wavefront, as in the single slit experiment.

They are both manifestations of the same phenomenon so the distinction is fairly arbitrary, but convenient.

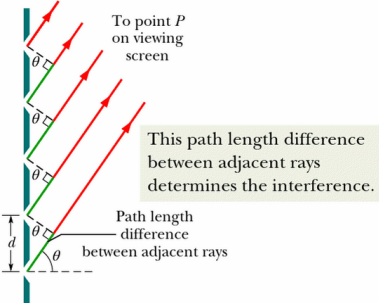
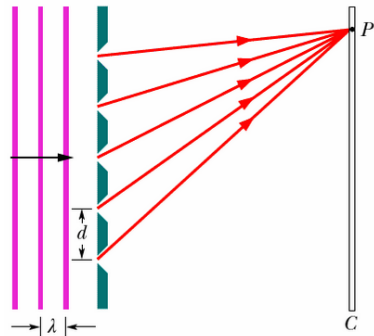
## Problem—double-slit experiment with diffraction from each slit included

In a double slit experiment, the wavelength  $\lambda$  of the light source is 405 nm, the slit separation  $d$  is  $19.44\ \mu\text{m}$ , and the slit width  $a$  is  $4.050\ \mu\text{m}$ . Consider the interference of the light from the two slits and also the diffraction of the light through each slit.

- (a) How many bright interference fringes are within the central peak of the diffraction envelope?
- (b) How many bright fringes are between the first two side peaks (on one side of the central maximum) of the diffraction envelope?



# Diffraction gratings—introduction



- By generalizing the double slit experiment to many more slits, you end up with a **diffraction grating**. The figure at left shows a diffraction grating with five slits (normally many thousands of slits would be used).
- The figure at bottom left shows how the slits act to produce interference at a distant screen, where the rays from the slits can be considered to be parallel. The path difference between adjacent rays is  $d \sin \theta$  and constructive interference will occur when the path difference is an integer number of wavelengths.
- The grating equation for maxima is

$$d \sin \theta = m\lambda \text{ for } m = 0, 1, 2, \dots$$

# Mini-quiz: single and double slits

## Double-slit envelope function



The first diffraction minima on the two sides of a double slit diffraction pattern happen to coincide with the fourth side bright fringes at a certain angle  $\theta$ .

(a) How many bright fringes are in the central diffraction envelope? HINT: don't forget to include the  $m = 0$  central bright fringe.

Answer =

(b) To shift the coincidence to the fifth side bright fringe, how should the slit separations be changed?

- ☐ Increased
- ☐ Decreased

(c) To make the shift by changing the slit widths instead of the slit separation, how should the widths be changed?

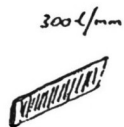
- ☐ Increased
- ☐ Decreased



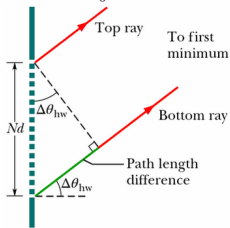
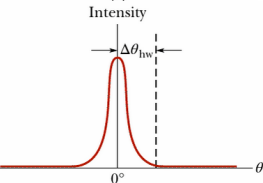
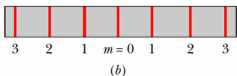
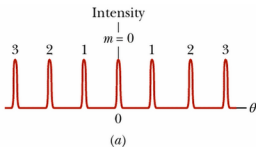
Submit for marking



# Demo: Lb05—diffraction gratings



# Diffraction gratings—width of the lines



- The top figure shows the intensity plot produced by a diffraction grating with a large number of slits illuminated by a laser. The bright fringes, called lines, are labeled with their order numbers  $m$ .
- Let's derive the half-width  $\Delta\theta_{\text{hw}}$  (see middle figure) for the  $m = 0$  line. By comparison with our derivation for a double slit, the path length difference from the top and bottom of a grating with  $N$  slits separated by distance  $d$  (see bottom figure) is  $\lambda$  for the first minimum. Therefore

$$Nd \sin \Delta\theta_{\text{hw}} = \lambda$$

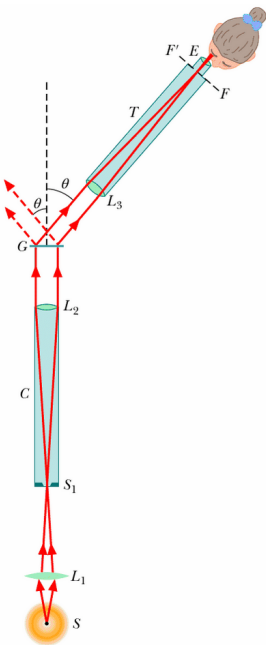
or, using the small angle approximation, the half-width of the central line is

$$\Delta\theta_{\text{hw}} = \frac{\lambda}{Nd}$$

- Generalising to a line at an angle  $\theta$  from the central axis, the equation becomes

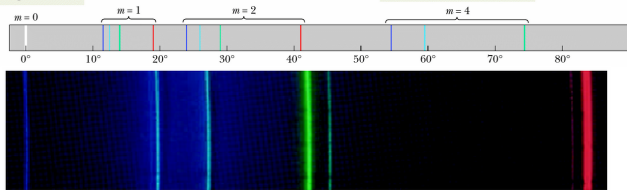
$$\Delta\theta_{\text{hw}} = \frac{\lambda}{Nd \cos \theta}$$

# Diffraction gratings—building a spectroscope



This is the center of the pattern.

The higher orders are spread out more in angle.



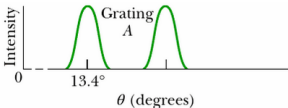
Department of Physics, Imperial College/Science Photo Library/  
Science Source

- Diffraction gratings are used in spectroscopes to analyze the light from objects such as stars. The figure at left shows a simple spectroscope. Light from a source  $S$  passes through some optics to a grating  $G$ , where the transmitted light is viewed by an observer through telescope  $T$ . By varying the angle  $\theta$  of the telescope we can determine the wavelength of lines emitted by the source.
- The figure at the top shows the spectrum produced by ionized hydrogen which has four discrete wavelengths in the visible range. The  $m = 3$  order is omitted for clarity since its lines overlap with the orders shown.
- The figure below the top one shows the emission lines of cadmium.

# Diffraction gratings—resolving power and dispersion

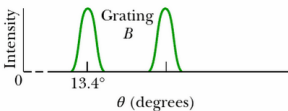
| Grating | $N$    | $d$ (nm) | $\theta$       | $D$ ( $^{\circ}/\mu\text{m}$ ) | $R$    |
|---------|--------|----------|----------------|--------------------------------|--------|
| A       | 10 000 | 2540     | $13.4^{\circ}$ | 23.2                           | 10 000 |
| B       | 20 000 | 2540     | $13.4^{\circ}$ | 23.2                           | 20 000 |
| C       | 10 000 | 1360     | $25.5^{\circ}$ | 46.3                           | 10 000 |

<sup>a</sup>Data are for  $\lambda = 589$  nm and  $m = 1$ .



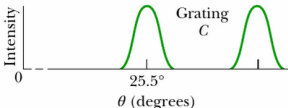
- The **dispersion**  $D$  of a grating is defined as

$$D = \frac{\Delta\theta}{\Delta\lambda}$$



- We can derive an expression for  $D$  from the location of the lines given by  $d \sin \theta = m\lambda$ . Taking differentials of this equation, we get  $d(\cos \theta)d\theta = m d\lambda$ . For small enough differences in  $\theta$  and  $\lambda$  we can write  $d(\cos \theta)\Delta\theta = m\Delta\lambda$ , and hence

$$D = \frac{\Delta\theta}{\Delta\lambda} = \frac{m}{d \cos \theta}$$



- To achieve high dispersion we must use a grating of smaller grating spacing  $d$  and/or use a higher order  $m$ . Note that the dispersion does not depend on the number of rulings in the grating.
- For a grating to distinguish two lines separated by  $\Delta\lambda$ , it must spread the lines apart by more than  $\Delta\theta_{\text{hw}}$ . If we substitute  $\Delta\theta_{\text{hw}}$  for  $\Delta\theta$  we have  $\lambda/N = m\Delta\lambda$ , and defining the **resolving power**  $R$  of a grating as  $\lambda/\Delta\lambda$ , we have

$$R = \frac{\lambda}{\Delta\lambda} = Nm$$

Lines observed with gratings from the table. Examine the differences carefully.

# Problem—dispersion and resolving power of a diffraction grating

A diffraction grating has  $1.26 \times 10^4$  rulings uniformly spaced over width  $w = 25.4$  mm. It is illuminated at normal incidence by yellow light from a sodium vapor lamp. This light contains two closely spaced emission lines (known as the sodium doublet) of wavelengths 589.00 nm and 589.59 nm.

- (a) At what angle does the first order maximum occur (on either side of the center of the diffraction pattern) for the wavelength of 589.00 nm?
- (b) Using the dispersion of the grating, calculate the angular separation between the two lines in the first order.
- (c) What is the least number of rulings a grating can have and still be able to resolve the sodium doublet in the first order?

# Solution

## Sample Problem 36.6.1 Dispersion and resolving power of a diffraction grating

A diffraction grating has  $1.26 \times 10^4$  rulings uniformly spaced over width  $w = 25.4$  mm. It is illuminated at normal incidence by yellow light from a sodium vapor lamp. This light contains two closely spaced emission lines (known as the sodium doublet) of wavelengths 589.00 nm and 589.59 nm.

(a) At what angle does the first-order maximum occur (on either side of the center of the diffraction pattern) for the wavelength of 589.00 nm?

### KEY IDEA

The maxima produced by the diffraction grating can be determined with Eq. 36.5.1 ( $d \sin \theta = m\lambda$ ).

**Calculations:** The grating spacing  $d$  is

$$\begin{aligned} d &= \frac{w}{N} = \frac{25.4 \times 10^{-3} \text{ m}}{1.26 \times 10^4} \\ &= 2.016 \times 10^{-6} \text{ m} = 2016 \text{ nm}. \end{aligned}$$

The first-order maximum corresponds to  $m = 1$ . Substituting these values for  $d$  and  $m$  into Eq. 36.5.1 leads to

$$\begin{aligned} \theta &= \sin^{-1} \frac{m\lambda}{d} = \sin^{-1} \frac{(1)(589.00 \text{ nm})}{2016 \text{ nm}} \\ &= 16.99^\circ \approx 17.0^\circ. \end{aligned} \quad (\text{Answer})$$

(b) Using the dispersion of the grating, calculate the angular separation between the two lines in the first order.

### KEY IDEAS

(1) The angular separation  $\Delta\theta$  between the two lines in the first order depends on their wavelength difference  $\Delta\lambda$  and the dispersion  $D$  of the grating, according to Eq. 36.6.1 ( $D = \Delta\theta/\Delta\lambda$ ). (2) The dispersion  $D$  depends on the angle  $\theta$  at which it is to be evaluated.

**Calculations:** We can assume that, in the first order, the two sodium lines occur close enough to each other for us to evaluate  $D$  at the angle  $\theta = 16.99^\circ$  we found in part (a) for one of those lines. Then Eq. 36.6.2 gives the dispersion as

$$\begin{aligned} D &= \frac{m}{d \cos \theta} = \frac{1}{(2016 \text{ nm})(\cos 16.99^\circ)} \\ &= 5.187 \times 10^{-4} \text{ rad/nm}. \end{aligned}$$

From Eq. 36.6.1 and with  $\Delta\lambda$  in nanometers, we then have

$$\begin{aligned} \Delta\theta &= D \Delta\lambda = (5.187 \times 10^{-4} \text{ rad/nm})(589.59 - 589.00) \\ &= 3.06 \times 10^{-4} \text{ rad} = 0.0175^\circ. \end{aligned} \quad (\text{Answer})$$

You can show that this result depends on the grating spacing  $d$  but not on the number of rulings there are in the grating.

(c) What is the least number of rulings a grating can have and still be able to resolve the sodium doublet in the first order?

### KEY IDEAS

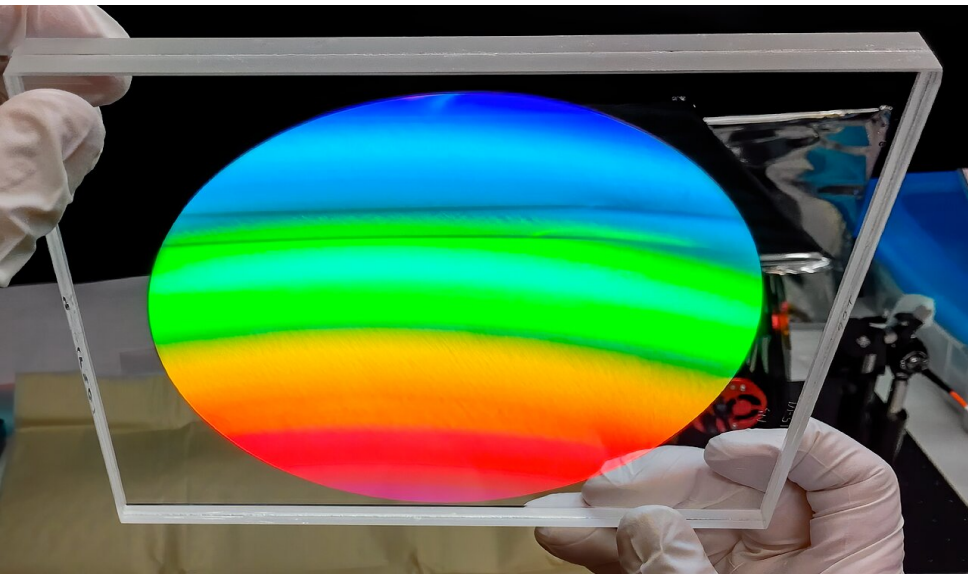
(1) The resolving power of a grating in any order  $m$  is physically set by the number of rulings  $N$  in the grating according to Eq. 36.6.4 ( $R = Nm$ ). (2) The smallest wavelength difference  $\Delta\lambda$  that can be resolved depends on the average wavelength involved and on the resolving power  $R$  of the grating, according to Eq. 36.6.3 ( $R = \lambda_{\text{avg}}/\Delta\lambda$ ).

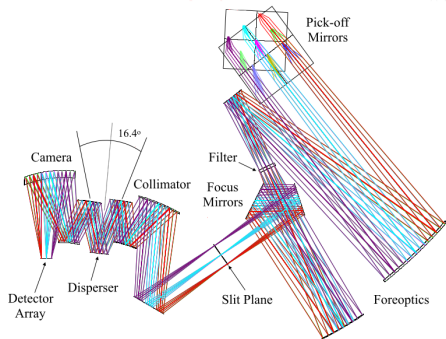
**Calculation:** For the sodium doublet to be barely resolved,  $\Delta\lambda$  must be their wavelength separation of 0.59 nm, and  $\lambda_{\text{avg}}$  must be their average wavelength of 589.30 nm. Thus, we find that the smallest number of rulings for a grating to resolve the sodium doublet is

$$\begin{aligned} N &= \frac{R}{m} = \frac{\lambda_{\text{avg}}}{m \Delta\lambda} \\ &= \frac{589.30 \text{ nm}}{(1)(0.59 \text{ nm})} = 999 \text{ rulings}. \end{aligned} \quad (\text{Answer})$$

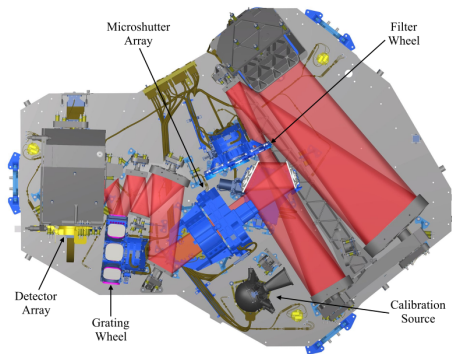


# A VPH grating used for the DESI galaxy survey at Kitt Peak





**Fig. 1.** Optical path through the NIRSpec instrument. The dispersion direction is out of the image.

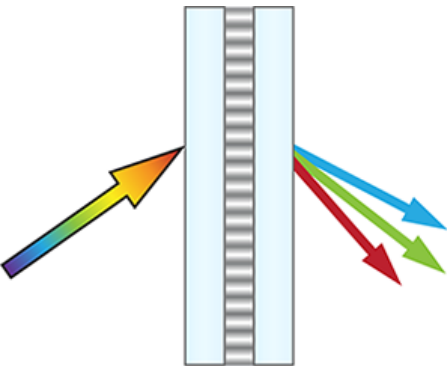


**Fig. 2.** CAD rendering of NIRSpec with its three major mechanisms and the calibration source identified.

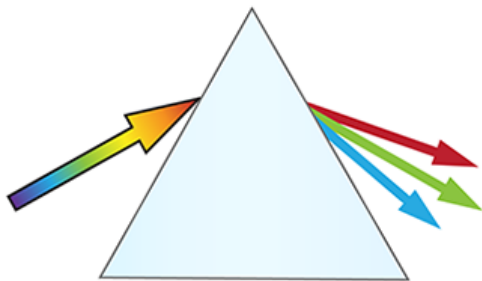
“NIRSpec employs two sets of three diffraction gratings providing spectral resolutions of  $R \approx 1000$  and  $R \approx 2700$ , respectively. The two sets of gratings are formally specified to cover the  $1.0\text{--}5.0\mu\text{m}$  wavelength region in each of three overlapping bands:  $1.0\text{--}1.8\mu\text{m}$  (Band I);  $1.7\text{--}3.0\mu\text{m}$  (Band II) and  $2.9\text{--}5.0\mu\text{m}$  (Band III), with the blaze wavelength of each grating set to the harmonic mean of the above band endpoints to balance the blaze function across the band. These three nominal bands are common to the  $R \approx 1000$  and  $R \approx 2700$  gratings, whose properties are listed in Table 3. The gratings measure  $70\text{ mm} \times 70\text{ mm}$  in active area and were manufactured by Carl Zeiss Optronics by mechanically ruling triangular grooves having the desired blaze angle directly onto a gold layer deposited on a Zerodur substrate. The resulting gratings all have very high peak efficiencies ( $> 80\%$ ) and low straylight.”

# Diffraction gratings cf. prisms

Diffraction gratings deflect longer wavelengths by a greater angle than short wavelengths. Prisms do the opposite—which is a result of the variation of refractive index with  $\lambda$ , which in turn is due to the longer wavelengths getting less of a “phase kick” from each thin layer of glass, which in turn is related to how close the frequency of the light is to the natural resonant frequency of the oscillating electrons in the glass.



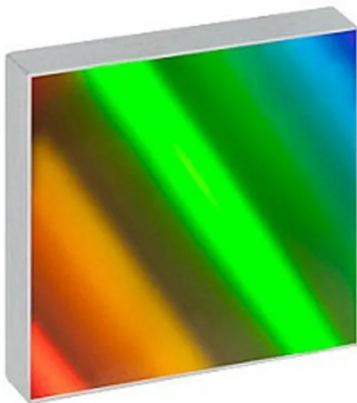
VPH Grating



Prism

# Mini-quiz: diffraction gratings

## Resolving power of a diffraction grating



Same qrcode as before

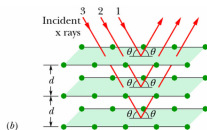
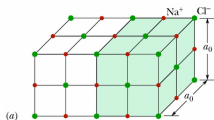
If we cover half a diffraction grating with opaque tape, what happens to the resolving power  $R$  of the grating?

- ☐  $R$  increases
- ☐  $R$  decreases

Submit for marking



# X-ray diffraction from NaCl crystals



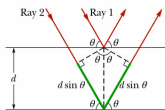
- X-rays have wavelengths of  $\sim 0.1$  nm, about  $1/5000$  that of visible light. Since x-rays are about the same size as atomic dimensions, it is impractical to make x-ray diffraction gratings by mechanical means. However, by using the natural repetitive structures of crystals such as NaCl (rock salt), it is possible to deflect x-rays much like a diffraction grating.

- The panel (a) in the figure shows a **unit cell** of an NaCl crystal.

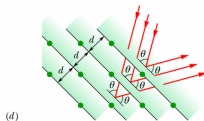
- Various planes of atoms can be found in the crystal, and these can effectively act as though they were reflecting the x-rays as shown in panel (b) when the path difference between rays from adjacent planes (see panel (c)) is given by Bragg's Law

$$2d \sin \theta = m\lambda \text{ for } m = 1, 2, 3, \dots$$

- Panel (d) shows an alternative array of planes in the NaCl crystal, which will also produce Bragg diffraction.

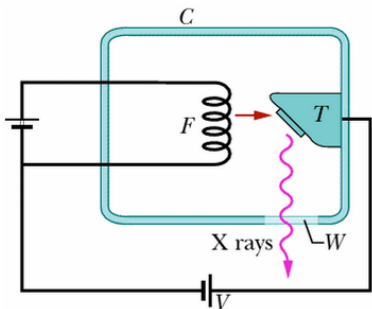


The extra distance of ray 2 determines the interference.



By convention in x-ray crystallography, the angle  $\theta$  is measured from the plane rather than the normal.

# X-ray diffraction—the diffraction equation



■ A beam of x-rays can be generated when electrons leaving heated filament  $F$  are accelerated through a potential difference  $V$  and strike a metal target  $T$ . The window  $W$  in the evacuated chamber  $C$  is transparent to x-rays.

■ By studying the angles through which the x-rays are diffracted by the various planes in a crystal, it is possible to deduce the unit cell structure.

■ The bottom figure shows one such set of planes for NaCl, separated by distance  $d$ . It can be easily shown to be  $d = a_0/\sqrt{20}$  where  $a_0$  is the length of the side of a unit cell.

