

How many of you have ever seen Mercury?

Now is a good time to look for it, since it reached its greatest elongation from the Sun a few days ago. It is easily visible in the evening sky 25 minutes after sunset (sunset is at 7:21pm tonight).

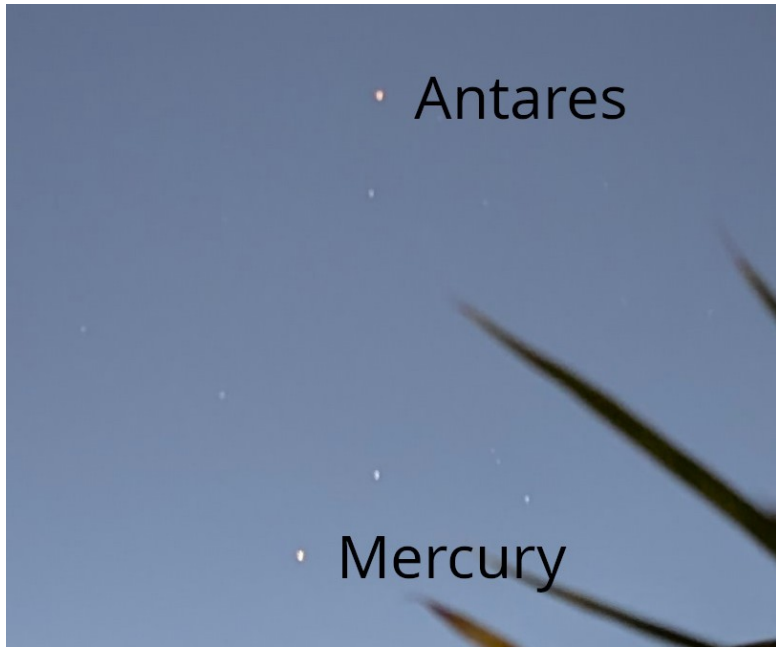


The Stellarium app

Mercury last night at 8:13pm



Mercury last night at 8:13pm



Lecture 12—learning outcomes

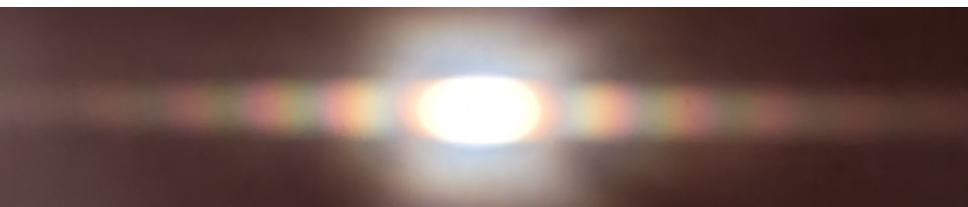
- Determine when a phase change will occur in light reflected from a boundary.
- For transparent films determine which thicknesses will lead to constructive and destructive interference for different wavelength of light.
- Draw a diagram showing a Lloyd mirror and a Michelson Interferometer. Explain how they work.
- Describe how the observation will change if a material with refractive index n is inserted into one of the arms/paths.
- Draw a diagram showing the diffraction pattern when light passes through a single slit. Show where maximum and minimum intensities will be observed. Write down an equation to describe this.
- Perform calculations to relate the intensity of light at the centre of a diffraction pattern to the intensity of light further along the screen.
- Describe how phasors can be used to show how the intensity of the light varies along the screen for light passing through a single slit.

Textbook chapters: 35.4–5, 36.1–2 <https://ebookcentral.proquest.com/lib/unsw/reader.action?c=UERG&docID=30765671&ppg=865>

From last lecture: Young's double slit experiment—under construction on Monday



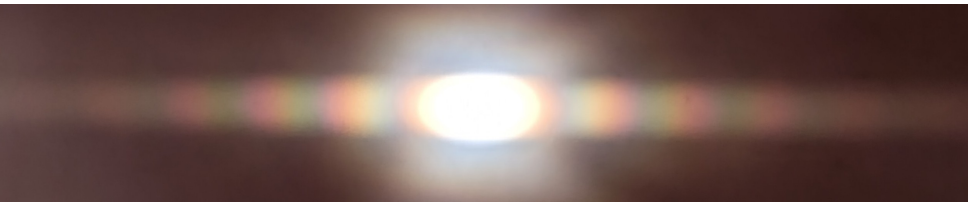
Young's double slit experiment demo—the results from yesterday



Slit width 0.04mm, separation 0.5mm

Looks good! But can we do better with a smaller slit separation? And there is a niggling worry: sunlight is only correlated over perpendicular distances of 0.030mm, as we showed in the last lecture, and our slit separation is about 16 times too big. So what is going on?

Young's double slit experiment demo—the results from yesterday



Slit width 0.04mm, separation 0.5mm

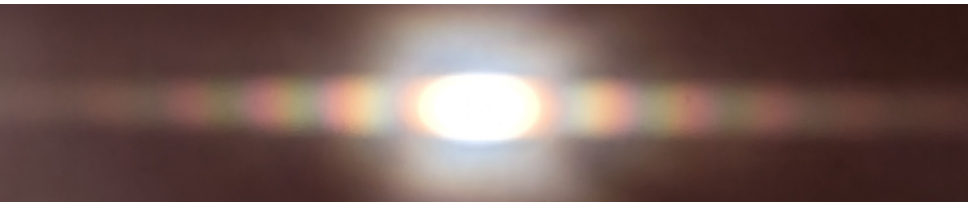


Slit width 0.15mm, separation 0.36mm; same scale

Strange, the smaller slit separation in the bottom image produces fringes that are closer together. That shouldn't happen!

Aside: the new slits are longer than the old ones, which explains the change in the vertical extent.

Young's double slit experiment demo—the results from yesterday



Slit width 0.04mm, separation 0.5mm



Slit width 0.15mm, separation 0.36mm; same scale

Explanation: the coloured fringes are produced by single-slit diffraction! The double-slit fringes are much closer together (by the ratio of slit width over separation) and won't be seen anyway since sunlight isn't correlated at the slit separations we used. So, mystery solved, and Veritasium's video is not a demonstration of double slit interference. Revisit the simulation from the last lecture:

https://javalab.org/en/double_slit_en/

Young's double slit experiment—Veritasium's slits



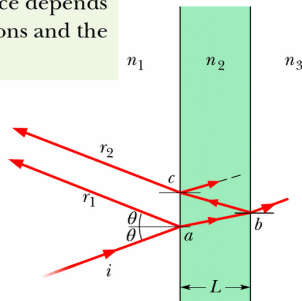
Slit width $\sim 0.12\text{mm}$, separation 0.20mm

However, Veritasium's slits do still appear to be too wide to show even single-slit interference.
Hm... maybe there is still a mystery. . . .

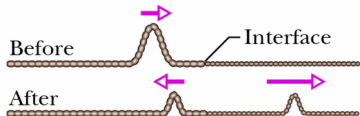
Interference from thin ($L \sim \lambda$) films

- The figure shows a ray i incident on a thin film of thickness L .
- Some of the light is reflected from the front surface, producing ray r_1 .
- Some passes into the thin film and is reflected from the back surface, emerging as ray r_2 .
- r_1 and r_2 are parallel, and coherent (provided the coherence length is greater than the path difference between them), the rays therefore interfere.
- To calculate the intensity of the resulting wavefront, we need to calculate the phase delay between r_1 and r_2 . This is a combination of the path length in the film multiplied by the refractive index n_2 (i.e., the OPL, the optical path length), and any phase changes that r_1 and r_2 undergo on reflection.

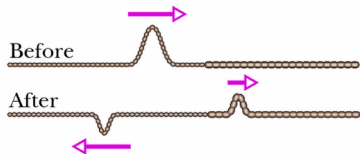
The interference depends on the reflections and the path lengths.



The phase change at an interface



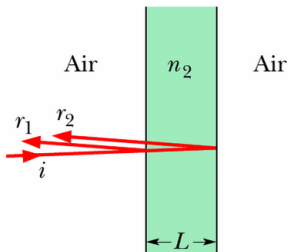
(a)



(b)

- How light behaves at a refractive index interface, is similar to how a pulse on a string behaves at a density interface.
- A pulse on a string speeds up when the linear density decreases.
- The top panel of the figure shows a pulse on a string approaching an interface with decrease in linear density. Part of the pulse continues on with no change in phase, part of the pulse is reflected, also with no change in phase.
- The bottom panel shows a pulse approaching a region of higher linear density. Once again the fraction of the pulse that continues to the right has no phase change, but in this case the reflected pulse has a phase change of π radians, or 0.5λ .
- Light behaves analogously: there is no phase change on reflection when n drops (and the speed of light in the medium increases); there is a phase change of π radians when n increases.

Reflections from a thin film in air



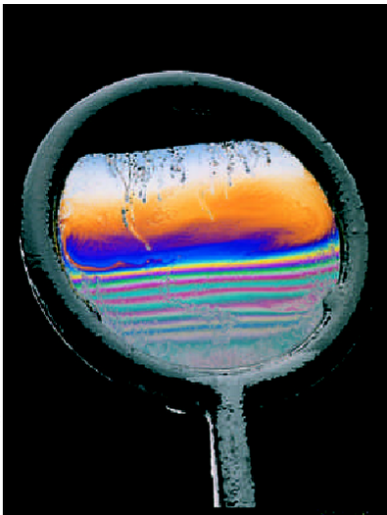
- For simplicity, let's consider a thin film with the incident light almost perpendicular to the surface, as shown in the figure. The length of the path of ray r_2 in the film is very close to $2L$.
- r_1 is a reflection from a low to high n interface, so it has a phase shift of π radians. r_2 has no phase shift from reflection, but travels the equivalent of $2n_2L$ further.
- For r_1 and r_2 to produce a bright reflection (and hence minimal transmission into the film), we have

$$\frac{2n_2L}{\lambda} = m + \frac{1}{2} \quad \text{for } m = 0, 1, 2, \dots$$

- For r_1 and r_2 to produce a minimal reflection (and hence maximum transmission into the film), we have

$$\frac{2n_2L}{\lambda} = m \quad \text{for } m = 0, 1, 2, \dots$$

Reflections from a very thin ($L \ll \lambda$) film in air



- If a film is so thin that $L \ll \lambda$, then the path length difference $2L$ can be neglected and we only have to consider the reflection phase shifts.
- The figure shows the reflection of light from a soapy water film spanning a vertical loop. The top portion of the soap film is very thin due to gravitational slumping. The π radian phase shift between the two reflected beams here causes them to destructively interfere. All the light is transmitted, making this portion appear dark.
- The colored interference fringes are caused by various wavelengths reaching peak reflectance from the film as L gradually increases towards the bottom of the loop.

Problem—thin-film interference of a water film in air

White light, with a uniform intensity across the visible wavelength range of 400 to 690 nm, is perpendicularly incident on a water film of refractive index $n_2 = 1.33$ and thickness $L = 320$ nm. The film is suspended in air. At what wavelength λ is the light reflected by the film brightest to an observer?

Solution

Sample Problem 35.4.1 Thin-film interference of a water film in air

White light, with a uniform intensity across the visible wavelength range of 400 to 690 nm, is perpendicularly incident on a water film, of index of refraction $n_2 = 1.33$ and thickness $L = 320$ nm, that is suspended in air. At what wavelength λ is the light reflected by the film brightest to an observer?

KEY IDEA

The reflected light from the film is brightest at the wavelengths λ for which the reflected rays are in phase with one another. The equation relating these wavelengths λ to the given film thickness L and film index of refraction n_2 is either Eq. 35.4.6 or Eq. 35.4.7, depending on the reflection phase shifts for this particular film.

Calculations: To determine which equation is needed, we should fill out an organizing table like Table 35.4.1. However, because there is air on both sides of the water film, the situation here is exactly like that in Fig. 35.4.3, and thus the table would be exactly like Table 35.4.1.

Then from Table 35.4.1, we see that the reflected rays are in phase (and thus the film is brightest) when

$$2L = \frac{\text{odd number}}{2} \times \frac{\lambda}{n_2},$$

which leads to Eq. 35.4.6:

$$2L = (m + \frac{1}{2}) \frac{\lambda}{n_2}.$$

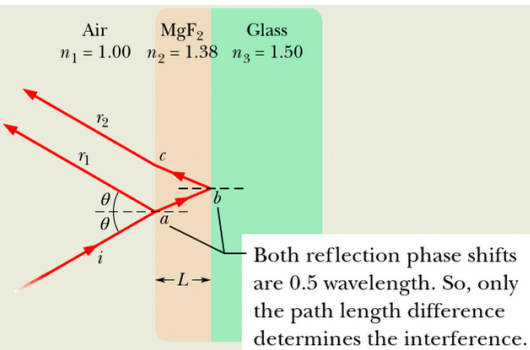
Solving for λ and substituting for L and n_2 , we find

$$\lambda = \frac{2n_2L}{m + \frac{1}{2}} = \frac{(2)(1.33)(320 \text{ nm})}{m + \frac{1}{2}} = \frac{851 \text{ nm}}{m + \frac{1}{2}}.$$

For $m = 0$, this gives us $\lambda = 1700$ nm, which is in the infrared region. For $m = 1$, we find $\lambda = 567$ nm, which is yellow-green light, near the middle of the visible spectrum. For $m = 2$, $\lambda = 340$ nm, which is in the ultraviolet region. Thus, the wavelength at which the light seen by the observer is brightest is

$$\lambda = 567 \text{ nm.} \quad (\text{Answer})$$

Problem—thin-film coating on a glass lens



In the figure, a glass camera lens is coated on one side with a thin film of magnesium fluoride (MgF₂) to reduce reflection from the lens surface so that more of the light enters the camera. The index of refraction of MgF₂ is 1.38; that of the glass is 1.50. What is the least coating thickness that eliminates (via interference) the reflections at the middle of the visible spectrum ($\lambda = 550 \text{ nm}$)? Assume that the light is approximately perpendicular to the lens surface.

Solution—part 1

Sample Problem 35.4.2 Thin-film interference of a coating on a glass lens

In Fig. 35.4.8, a glass camera lens is coated on one side with a thin film of magnesium fluoride (MgF_2) to reduce reflection from the lens surface so that more of the light enters the camera. The index of refraction of MgF_2 is 1.38; that of the glass is 1.50. What is the least coating thickness that eliminates (via interference) the reflections at the middle of the visible spectrum ($\lambda = 550 \text{ nm}$)? Assume that the light is approximately perpendicular to the lens surface.

KEY IDEA

Reflection is eliminated if the film thickness L is such that light waves reflected from the two film interfaces are exactly out of phase. The equation relating L to the given wavelength λ and the index of refraction n_2 of the thin film is either Eq. 35.4.6 or Eq. 35.4.7, depending on the reflection phase shifts at the interfaces.

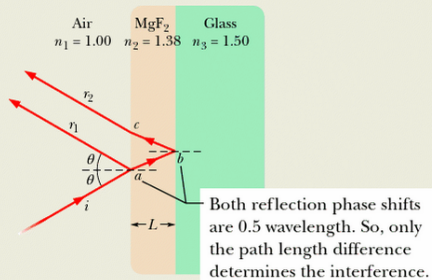


Figure 35.4.8 Unwanted reflections from glass can be suppressed (at a chosen wavelength) by coating the glass with a thin transparent film of magnesium fluoride of the properly chosen thickness.

Solution—part 2

Calculations: To determine which equation is needed, we fill out an organizing table like Table 35.4.1. At the first interface, the incident light is in air, which has a lesser index of refraction than the MgF_2 (the thin film). Thus, we fill in 0.5 wavelength under r_1 in our organizing table (meaning that the waves of ray r_1 are shifted by 0.5λ at the first interface). At the second interface, the incident light is in the MgF_2 , which has a lesser index of refraction than the glass on the other side of the interface. Thus, we fill in 0.5 wavelength under r_2 in our table.

Because both reflections cause the same phase shift, they tend to put the waves of r_1 and r_2 in phase. Since we want those waves to be *out of phase*, their path length difference $2L$ must be an odd number of half-wavelengths:

$$2L = \frac{\text{odd number}}{2} \times \frac{\lambda}{n_2}.$$

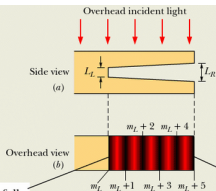
This leads to Eq. 35.4.6 (for a bright film sandwiched in air but for a dark film in the arrangement here). Solving that equation for L then gives us the film thicknesses that will eliminate reflection from the lens and coating:

$$L = (m + \frac{1}{2}) \frac{\lambda}{2n_2}, \quad \text{for } m = 0, 1, 2, \dots \quad (35.4.8)$$

We want the least thickness for the coating—that is, the smallest value of L . Thus, we choose $m = 0$, the smallest possible value of m . Substituting it and the given data in Eq. 35.4.8, we obtain

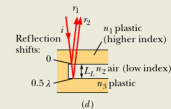
$$L = \frac{\lambda}{4n_2} = \frac{550 \text{ nm}}{(4)(1.38)} = 99.6 \text{ nm}. \quad (\text{Answer})$$

Problem—thin-film interference of a transparent wedge



This dark fringe is due to fully destructive interference. So, the reflected rays must be *out of phase*.

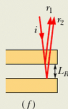
Here too, the dark fringe means that the reflected waves are *out of phase*.



The path length difference (down and back up) is $2L$.

Total reflection shift = 0.5 wavelength. So, the reflections put the waves out of phase.

We want the reflected waves to be out of phase. They already are out of phase because of the reflection shifts. So, we don't want the path length difference $2L$ to change that. Thus, $2L = (\text{integer})\lambda/n_2$.



The path length difference is $2L$ here too but the L is larger.

Here again, the waves are already out of phase by the reflection shifts. So, the path length difference must be $2L = (\text{integer})\lambda/n_2$, but with the larger L .

(e) Organizing Table

	n_1	n_2
Reflection phase shifts	0	0.5 wavelength
Path length difference	$2L$	

The top of the figure shows a transparent plastic block with a thin wedge of air at the right. The wedge thickness is exaggerated in the figure. A broad beam of red light with wavelength $\lambda = 632.8 \text{ nm}$, is directed downwards through the top of the block at an incident angle of 0° . Some of the light that passes into the plastic is reflected back up from the top and bottom surfaces of the wedge, which acts as a thin film (of air) with a thickness that varies uniformly and gradually from L_L at the left hand end to L_R at the right hand end. The plastic layers above and below the wedge of air are too thick to act as thin films. An observer looking down on the block sees an interference pattern consisting of six dark fringes and five bright red fringes along the wedge. What is the change of thickness $\Delta L = L_R - L_L$ along the wedge?

Solution—part 1

Sample Problem 35.4.3 Thin-film interference of a transparent wedge

Figure 35.4.9a shows a transparent plastic block with a thin wedge of air at the right. (The wedge thickness is exaggerated in the figure.) A broad beam of red light, with wavelength $\lambda = 632.8 \text{ nm}$, is directed downward through the top of the block (at an incidence angle of 0°). Some of the light that passes into the plastic is reflected back up from the top and bottom surfaces of the wedge, which acts as a thin film (of air) with a thickness that varies uniformly and gradually from L_L at the left-hand end to L_R at the right-hand end. (The plastic layers above and below the wedge of air are too thick to act as thin films.) An observer looking down on the block sees an interference pattern consisting of six dark fringes and five bright red fringes along the wedge. What is the change in thickness $\Delta L (= L_R - L_L)$ along the wedge?

KEY IDEAS

- (1) The brightness at any point along the left–right length of the air wedge is due to the interference of the waves reflected at the top and bottom interfaces of the wedge.
- (2) The variation of brightness in the pattern of bright and dark fringes is due to the variation in the thickness of the wedge. In some regions, the thickness puts the reflected waves in phase and thus produces a bright reflection (a bright red fringe). In other regions, the thickness puts the reflected waves out of phase and thus produces no reflection (a dark fringe).

Organizing the reflections: Because the observer sees more dark fringes than bright fringes, we can assume that a dark fringe is produced at both the left and right ends

of the wedge. Thus, the interference pattern is that shown in Fig. 35.4.9b.

We can represent the reflection of light at the top and bottom interfaces of the wedge, at any point along its length, with Fig. 35.4.9c, in which L is the wedge thickness at that point. Let us apply this figure to the left end of the wedge, where the reflections give a dark fringe.

We know that, for a dark fringe, the waves of rays r_1 and r_2 in Fig. 35.4.9d must be out of phase. We also know that the equation relating the film thickness L to the light's wavelength λ and the film's index of refraction n_2 is either Eq. 35.4.6 or Eq. 35.4.7, depending on the reflection phase shifts. To determine which equation gives a dark fringe at the left end of the wedge, we should fill out an organizing table like Table 35.4.1, as shown in Fig. 35.4.9e.

At the top interface of the wedge, the incident light is in the plastic, which has a greater n than the air beneath that interface. So, we fill in 0 under r_1 in our organizing table. At the bottom interface of the wedge, the incident light is in air, which has a lesser n than the plastic beneath that interface. So we fill in 0.5 wavelength under r_2 . So, the phase difference due to the reflection shifts is 0.5 wavelength. Thus the reflections alone tend to put the waves of r_1 and r_2 out of phase.

Reflections at left end (Fig. 35.4.9d): Because we see a dark fringe at the left end of the wedge, which the reflection phase shifts alone would produce, we don't want the path length difference to alter that condition. So, the path length difference $2L$ at the left end must be given by

$$2L = \text{integer} \times \frac{\lambda}{n_2},$$

Solution—part 2

which leads to Eq. 35.4.7:

$$2L = m \frac{\lambda}{n_2}, \quad \text{for } m = 0, 1, 2, \dots \quad (35.4.9)$$

Reflections at right end (Fig. 35.4.9f): Equation 35.4.9 holds not only for the left end of the wedge but also for any point along the wedge where a dark fringe is observed, including the right end, with a different integer value of m for each fringe. The least value of m is associated with the least thickness of the wedge where a dark fringe is observed. Progressively greater values of m are associated with progressively greater thicknesses of the wedge where a dark fringe is observed. Let m_L be the value at the left end. Then the value at the right end must be $m_L + 5$

because, from Fig. 35.4.9b, the right end is located at the fifth dark fringe from the left end.

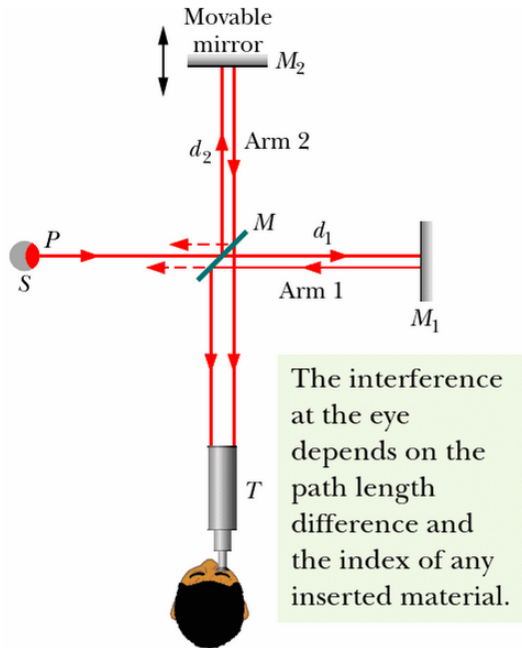
Thickness difference: To find ΔL , we first solve Eq. 35.4.9 twice—once for the thickness L_L at the left end and once for the thickness L_R at the right end:

$$L_L = (m_L) \frac{\lambda}{2n_2}, \quad L_R = (m_L + 5) \frac{\lambda}{2n_2}. \quad (35.4.10)$$

We can now subtract L_L from L_R and substitute $n_2 = 1.00$ for the air within the wedge and $\lambda = 632.8 \times 10^{-9} \text{ m}$:

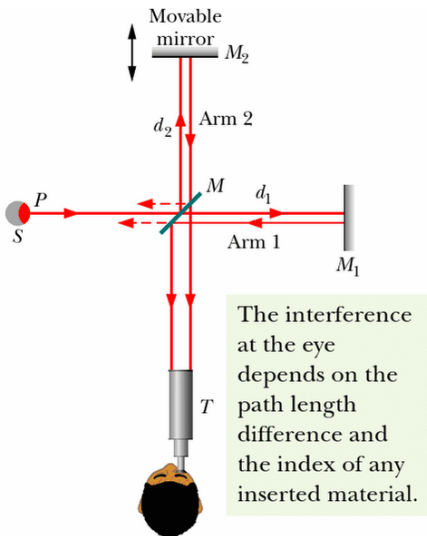
$$\begin{aligned} \Delta L = L_R - L_L &= \frac{(m_L + 5)\lambda}{2n_2} - \frac{m_L\lambda}{2n_2} = \frac{5}{2} \frac{\lambda}{n_2} \\ &= 1.58 \times 10^{-6} \text{ m}. \end{aligned} \quad (\text{Answer})$$

Michelson's interferometer



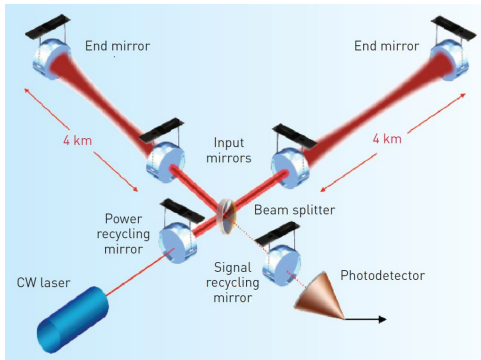
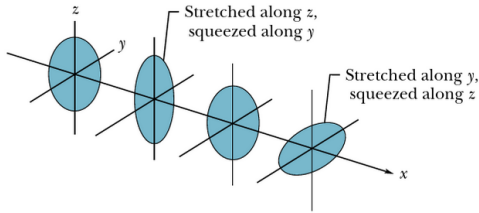
An interferometer is a device that can be used to measure lengths, or changes in length, with great accuracy by means of interference fringes. The figure shows Michelson's interferometer. Rays from the source S reach the beam splitter M which splits the light into two beams which reflect from mirrors M_1 and M_2 back to M and then to telescope T . An observer sees a pattern of interference fringes in the telescope.

Michelson's interferometer



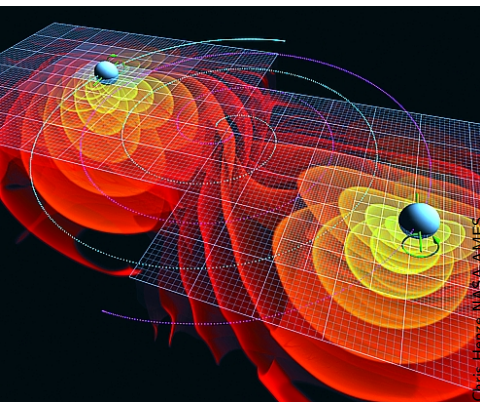
- If we insert a piece of thin transparent material into one of the beams, say in Arm 1, then the optical path difference caused by traversing this twice is $2L(n - 1)$, which causes a phase change (in wavelengths) of $2L(n - 1)/\lambda$.
- Each phase change of one wavelength causes a shift of one fringe in the pattern. So by counting the number of fringes that the pattern has moved, we can determine L if we know n , or alternatively, n if we know L .

Detecting gravitational waves using Michelson's interferometer



When a pair of massive astronomical bodies spiral into each other, their motion produces gravitational waves, which travel outwards at the speed of light. The waves alternately stretch and squeeze space itself. Gravitational waves were predicted by Albert Einstein in 1916. In 1972, Rainer Weiss at MIT suggested that these waves may be detectable using a version of Michelson's interferometer. This proposal led to the development of the Laser Interferometer Gravitational wave Observatory (LIGO) in Louisiana; others followed around the world. Each arm of LIGO consists of evacuated tubes 4 km long. In 2015, LIGO detected a gravitational wave. Computations revealed that it came from the merger of two black holes, one with a mass of 29 solar masses and the other of 36 solar masses, at a distance of 1.3 billion light years. See **Veritasium's excellent video on the subject:** <https://www.youtube.com/watch?v=iphcyNWFD1Q>

Gravitational wave “inspiral”



Gravity waves emitted from two black holes

- Any orbiting system emits gravitational waves.
- In the case of the Earth orbiting the Sun, the energy loss is about 200 watts. At this rate it will take 1×10^{13} times the current age of the Universe for the Earth to spiral into the Sun.
- The energy loss rate can be much large, e.g., a pair of solar-mass neutron stars separated by 189,000 km will orbit each other every 17 minutes and become closer by 0.1 km/year
- Once they reach a separation of 1890 km, their period will be 1 second and the orbit will shrink by 4 metres per revolution.
- They resulting gravitational waves will show a characteristic “chirp” signal as the stars merge to form a black hole.

Optical Observations of LIGO Source GW 170817 by the Antarctic Survey Telescopes at Dome A, Antarctica

Lei Hu, Xuefeng Wu, I. Andreoni, Michael C. B. Ashley, J. Cooke, Xiangqun Cui, Fujia Du, Zigao Dai, Bozhong Gu, Yi Hu, Haiping Lu, Xiaoyan Li, Zhengyang Li, Ensi Liang, Liangduan Liu, Bin Ma, Zhaohui Shang, Tianrui Sun, N. B. Suntzeff, Charling Tao, Lifan Wang, Xiaofeng Wang, Haikun Wen, Di Xiao, Jin Xu, Ji Yang, Shihai Yang, Xiangyan Yuan, Hui Zhang, and Jilin Zhou

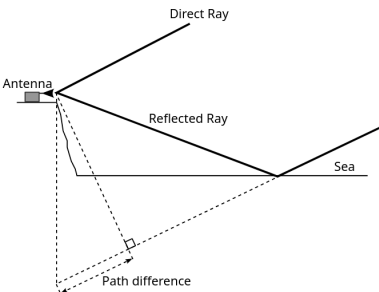
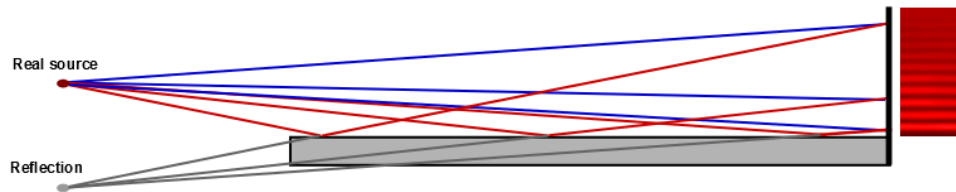
The event remains the only electromagnetic counterpart of a gravitational wave that has been detected. It was caused by the inspiral of two neutron stars, with a combined mass of about 2.8 solar masses. It is believed that a very massive neutron star was formed, just below the mass needed to form a black hole. The event produced large quantities of *r*-process elements, including about 10 Earth masses of gold and platinum.

Follow up of GW170817 and its electromagnetic counterpart by Australian-led observing programs

I. Andreoni, K. Ackley, J. Cooke, A. Acharyya, J. R. Allison, G. E. Anderson, M. C. B. Ashley, D. Baade, M. Bailes, K. Bannister, A. Beardsley, M. S. Bessell, F. Bian, P. A. Bland, M. Boer, T. Boller, A. Brandeker, I. S. Brown, D. A. H. Buckley, S.-W. Chang, D. M. Coward, S. Crawford, H. Crisp, B. Crosse, A. Cucchiara, M. Cupak, J. S. de Gois, A. Deller, H. A. R. Devillepoix, D. Dobie, E. Elmer, D. Emrich, W. Farah, T. J. Farrell, T. Franzen, B. M. Gaensler, D. K. Galloway, B. Gendre, T. Giblin, A. Goobar, J. Green, P. J. Hancock, B. A. D. Hartig, E. J. Howell, L. Horsley, A. Hotan, R. M. Howie, L. Hu, Y. Hu, C. W. James, S. Johnston, M. Johnston-Hollitt, D. L. Kaplan, M. Kasliwal, E. F. Keane, D. Kenney, A. Klotz, R. Lau, R. Laugier, E. Lenc, X. Li, E. Liang, C. Lidman, L. C. Luvaul, C. Lynch, B. Ma, D. Macpherson, J. Mao, D. E. McClelland, C. McCully, A. Möller, M. F. Morales, D. Morris, T. Murphy, K. Noysena, C. A. Onken, N. B. Orange, S. Osłowski, D. Pallot, J. Paxman, S. B. Potter, T. Pritchard, W. Raja, R. Ridden-Harper, E. Romero-Colmenero, E. M. Sadler, E. K. Sansom, R. A. Scalzo, B. P. Schmidt, S. M. Scott, N. Seghouani, Z. Shang, R. M. Shannon, L. Shao, M. M. Shara, R. Sharp, M. Sokolowski, J. Sollerman, J. Staff, K. Steele, T. Sun, N. B. Suntzeff, C. Tao, S. Tingay, M. C. Towner, P. Thierry, C. Trott, B. E. Tucker, P. Vaisanen, V. Venkatraman Krishnan, M. Walker, L. Wang, X. Wang, R. Wayth, M. Whiting, A. Williams, T. Williams, C. Wolf, C. Wu, X. Wu, J. Yang, X. Yuan, H. Zhang, J. Zhou, and H. Zovaro

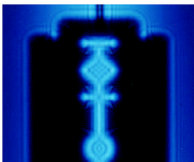
Multi-messenger Observations of a Binary Neutron Star Merger

Lloyd's Mirror



- In Lloyd's Mirror Experiment (see figure above), monochromatic light from a source bounces off a mirror at a grazing incidence angle and interferes with itself, thereby producing interference patterns.
- Australian radio astronomers used this technique in 1948 to measure the accurate positions of a few strong radio sources (see figure at left). They observed fringes as radio waves bounced off the sea as the sources rose, and obtained positions accurate to about 0.1° , which would otherwise have required a radio telescope with a diameter of 100 metres.

Diffraction from a single slit, and edges

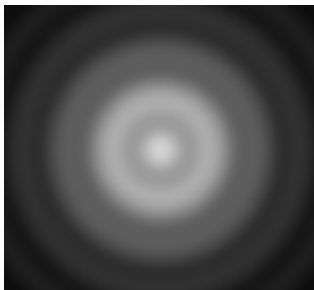


- In our earlier discussion of the double slit experiment, we glossed over the issue of diffraction from a single slit, showing it as simply a “flaring” of the oncoming wavefront.
- In fact, a single slit produces interference fringes on its own.
- The top figure shows the diffraction pattern from a single slit.
- The bottom figure shows the complex diffraction pattern from a razor blade.
- We will now show how a single slit can produce fringes, and derive their spacing.

Fresnel and Fraunhofer diffraction from a circular aperture



Fresnel diffraction

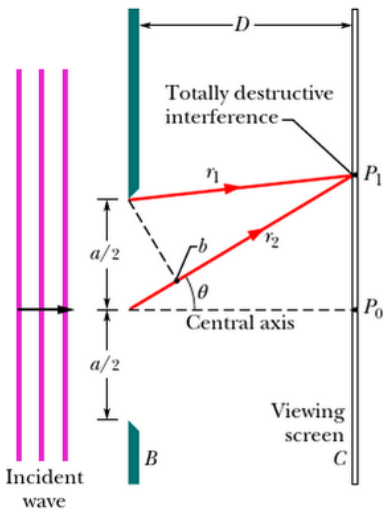


Fraunhofer diffraction

- Our analysis today will always be in the “far-field”, i.e., where the distance to the screen is much greater than the characteristic size of the diffracting object.
- The approximations that this allows result in a solution known as *Fraunhofer diffraction*.
- When you get close to the object the mathematics becomes more complicated, and is known as *Fresnel diffraction*.
- The images at left are part of a small movie that you can watch at the URL https://upload.wikimedia.org/wikipedia/commons/a/a3/Fresnel_to_Fraunhofer_transition.gif which shows the transition from Fresnel diffraction to Fraunhofer diffraction as the distance from a circular aperture increases.

Single slit diffraction

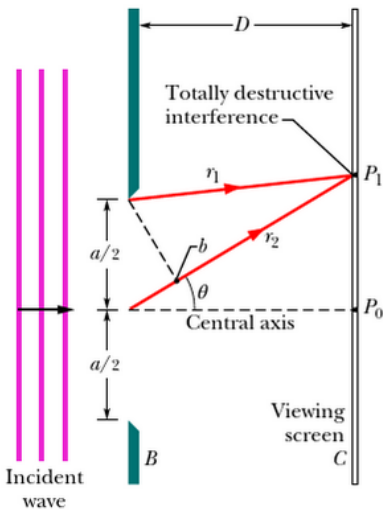
This pair of rays cancel each other at P_1 . So do all such pairings.



- To show how fringes can arise from single slit diffraction, we can apply Huygens' Principle that says that every point on the slit acts as a source of spherical wavefronts. To determine the intensity at a point P_1 on a screen (see the figure), we sum the wavefronts from all points in the slit.
- You may wonder why we covered the double slit first in lectures. The reason is that the double slit is relatively easy since we can make the approximation that it consists of two lines of point sources.
- If we approximated the single slit as one line of point sources we wouldn't see any interference.
- Single slit interference relies on a having a slit of finite width, of at least a wavelength or so.
- Finding the maximum and minimum for the single slit is fairly easy, as we will show in the next slide. However, finding the intensity is more difficult since we have to add multiple phasors, as compared to the double slit where we only had to add two.

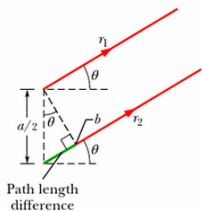
Single slit diffraction

This pair of rays cancel each other at P_1 . So do all such pairings.



- To show how fringes can arise from single slit diffraction, we can apply Huygens' Principle that says that every point on the slit acts as a source of spherical wavefronts. To determine the intensity at a point P_1 on a screen (see the figure), we sum the wavefronts from all points in the slit.
- Let's find the position of a dark fringe.
- To make the calculation easy, divide the slit into two halves as shown in the figure. Then draw a ray r_1 from the top of the slit to P_1 and another ray from the center of the slit to P_1 . Suppose the path difference between these two rays is $\lambda/2$, leading to destructive interference.
- Now shift the starting point of both rays down by a small amount, while still terminating at P_1 . Assuming that $D \gg a$, the path difference remains the same, and the rays persist in destructively interfering. Continue until you have covered the entire slit. It is clear that the net effect is a dark fringe.
- A similar argument works for bright fringes, and in fact any phase shift between r_1 and r_2 .

Single slit diffraction—the case $D \gg a$



This path length difference shifts one wave from the other, which determines the interference.

- For $D \gg a$, the two rays r_1 and r_2 are approximately parallel.
- The path length difference between the rays is $(a/2) \sin \theta$, and for the first dark fringe this equals $\lambda/2$. Generalising to the m^{th} dark fringe, the criterion is

$$a \sin \theta = m\lambda, \quad \text{for } m = 1, 2, 3, \dots$$

- Interestingly, there is no simple exact formula for the position of the bright fringes. While the bright fringes are approximately half-way between the dark ones, for an exact answer you have to differentiate the formula for intensity as a function of θ and set it to zero.

Problem—single slit diffraction pattern with white light

A slit of width a is illuminated by white light. Assume that each small wavelength range in the light is sufficiently coherent over the slit.

- (a) For what value of a will the first minimum for red light of wavelength $\lambda = 650 \text{ nm}$ appear at $\theta = 15^\circ$?
- (b) What is the wavelength λ' of the light whose first side diffraction *maximum* is at 15° , thus coinciding with the first minimum for the red light?

Solution—part 1

Sample Problem 36.1.1 Single-slit diffraction pattern with white light

A slit of width a is illuminated by white light.

(a) For what value of a will the first minimum for red light of wavelength $\lambda = 650$ nm appear at $\theta = 15^\circ$?

KEY IDEA

Diffraction occurs separately for each wavelength in the range of wavelengths passing through the slit, with the locations of the minima for each wavelength given by Eq. 36.1.3 ($a \sin \theta = m\lambda$).

Calculation: When we set $m = 1$ (for the first minimum) and substitute the given values of θ and λ , Eq. 36.1.3 yields

$$\begin{aligned} a &= \frac{m\lambda}{\sin \theta} = \frac{(1)(650 \text{ nm})}{\sin 15^\circ} \\ &= 2511 \text{ nm} \approx 2.5 \mu\text{m}. \end{aligned} \quad (\text{Answer})$$

For the incident light to flare out that much ($\pm 15^\circ$ to the first minima) the slit has to be very fine indeed—in this case, a mere four times the wavelength. For comparison, note that a fine human hair may be about $100 \mu\text{m}$ in diameter.

Solution—part 2

(b) What is the wavelength λ' of the light whose first side diffraction maximum is at 15° , thus coinciding with the first minimum for the red light?

KEY IDEA

The first side maximum for any wavelength is about halfway between the first and second minima for that wavelength.

Calculations: Those first and second minima can be located with Eq. 36.1.3 by setting $m = 1$ and $m = 2$, respectively. Thus, the first side maximum can be located *approximately* by setting $m = 1.5$. Then Eq. 36.1.3 becomes

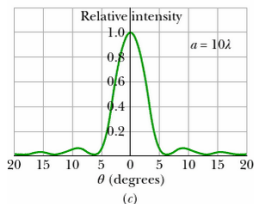
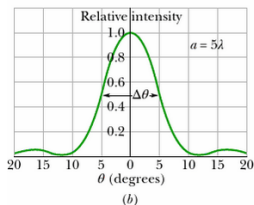
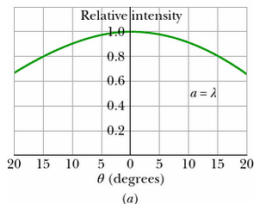
$$a \sin \theta = 1.5\lambda'.$$

Solving for λ' and substituting known data yield

$$\begin{aligned}\lambda' &= \frac{a \sin \theta}{1.5} = \frac{(2511 \text{ nm})(\sin 15^\circ)}{1.5} \\ &= 430 \text{ nm.} \quad (\text{Answer})\end{aligned}$$

Light of this wavelength is violet (far blue, near the short-wavelength limit of the human range of visible light). From the two equations we used, can you see that the first side maximum for light of wavelength 430 nm will always coincide with the first minimum for light of wavelength 650 nm, no matter what the slit width is? However, the angle θ at which this overlap occurs does depend on slit width. If the slit is relatively narrow, the angle will be relatively large, and conversely.

Single slit diffraction intensity as a function of a/λ



- The figure shows the relative intensity in single slit diffraction for three values of the ratio a/λ . The wider the slit is, the narrower the central diffraction maximum.
- The intensity of the pattern as a function of θ for single slit diffraction is (as we shall prove shortly),

$$I(\theta) = I_m \left(\frac{\sin \alpha}{\alpha} \right)^2$$

where

$$\alpha = \frac{\pi a}{\lambda} \sin \theta$$

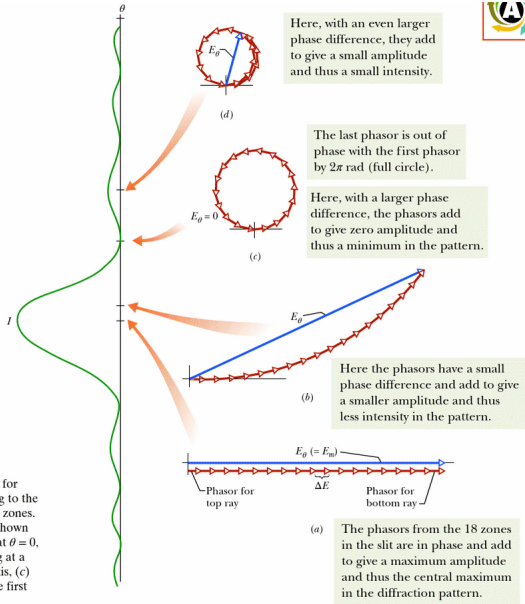
and I_m is the maximum intensity at $\theta = 0$.

- Inspection of the top formula shows that the minimum intensity occurs at $\alpha = m\pi$, for $m = 1, 2, 3, \dots$. Substituting this into the bottom formula gives

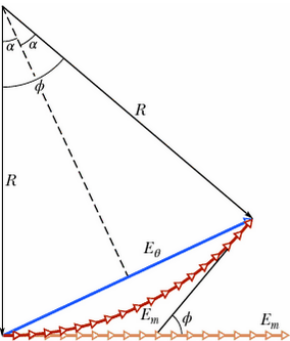
$$a \sin \theta = m\lambda, \quad \text{for } m = 1, 2, 3, \dots$$

as we had before.

Phasor diagrams for a single slit with 18 zones



Single slit diffraction intensity derivation



- The figure shows adding up the phasors for point (b) in the last slide.
- The angle ϕ is the phase difference resulting from adding up the 18 phasors shown. From the geometry,

$$\sin \frac{1}{2}\phi = \frac{E_0}{2R}$$

- Since the arc E_m subtends an angle ϕ at a radius of R ,

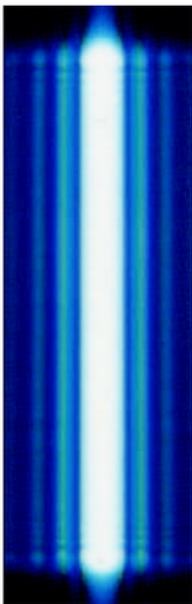
$$\phi = \frac{E_m}{R}$$

- Solving for R and substituting into the earlier formulae,

$$E_\theta = \frac{E_m}{\frac{1}{2}\phi} \sin \frac{1}{2}\phi$$

- Recalling that $I \propto E^2$, and noting $\alpha = \frac{1}{2}\phi$, we then reach the equations from two slides ago.

Problem—single slit diffraction intensities



Find the intensities of the first three secondary maxima (side maxima) in the single slit diffraction pattern in the figure measured as a percentage of the intensity of the central maximum. You should approximate the position of the maximum as halfway between the minima.

How does the result depend on the wavelength and the slit width?

Solution

Sample Problem 36.2.1 Intensities of the maxima in a single-slit interference pattern

Find the intensities of the first three secondary maxima (side maxima) in the single-slit diffraction pattern of Fig. 36.1.1, measured as a percentage of the intensity of the central maximum.

KEY IDEAS

The secondary maxima lie approximately halfway between the minima, whose angular locations are given by Eq. 36.2.4 ($\alpha = m\pi$). The locations of the secondary maxima are then given (approximately) by

$$a = \left(m + \frac{1}{2}\right)\pi, \quad \text{for } m = 1, 2, 3, \dots,$$

with α in radian measure. We can relate the intensity I at any point in the diffraction pattern to the intensity I_m of the central maximum via Eq. 36.2.2.

Calculations: Substituting the approximate values of α for the secondary maxima into Eq. 36.2.2 to obtain the relative intensities at those maxima, we get

$$\frac{I}{I_m} = \left(\frac{\sin \alpha}{\alpha}\right)^2 = \left(\frac{\sin(m + \frac{1}{2})\pi}{(m + \frac{1}{2})\pi}\right)^2, \quad \text{for } m = 1, 2, 3, \dots$$

The first of the secondary maxima occurs for $m = 1$, and its relative intensity is

$$\begin{aligned} \frac{I_1}{I_m} &= \left(\frac{\sin(1 + \frac{1}{2})\pi}{(1 + \frac{1}{2})\pi}\right)^2 = \left(\frac{\sin 1.5\pi}{1.5\pi}\right)^2 \\ &= 4.50 \times 10^{-2} \approx 4.5\%. \end{aligned} \quad (\text{Answer})$$

For $m = 2$ and $m = 3$ we find that

$$\frac{I_2}{I_m} = 1.6\% \quad \text{and} \quad \frac{I_3}{I_m} = 0.83\%. \quad (\text{Answer})$$

As you can see from these results, successive secondary maxima decrease rapidly in intensity. Figure 36.1.1 was deliberately overexposed to reveal them.

Scratch working space