

PHYS1231—part 2

- Your lecturer: Prof. Michael Ashley, m.ashley@unsw.edu.au.
- The best way to learn the physics for this course is to read the textbook and work through the problems.
- The webstream gives an excellent coverage of the content.
- My lectures will go beyond the elementary level. They will hopefully give you insights into the fundamental concepts, and have interesting side excursions.
- If there is a concept that you would really like me to cover, e-mail me.
- The examinable content is given by the learning outcomes and the indicated textbook sections. Non-examinable slides have a red dot at upper right.
- The slides are available here:
<https://mcba1.phys.unsw.edu.au/~mcba/PHYS1231/> including a version with a white background to help with printer ink.
- These notes are undergoing rapid change, so always grab the latest version.
- Several figures have been taken from *Electricity and Magnetism 3rd Ed.* by Purcell and Morin, which is an excellent textbook on the subject.

Lecture 9—learning outcomes

- State that an electromagnetic wave consists of oscillating electric and magnetic fields.
- Describe different parts of the electromagnetic spectrum, recognise that visible light is just a small part of this spectrum.
- Use the equations $E = E_m \sin(kx - \omega t)$ and $B = B_m \sin(kx - \omega t)$ to describe a wave traveling along the x axis.
- Relate the speed of light in a vacuum to the magnitude of the magnetic and electric fields using $c = \frac{E}{B} = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$
- Calculate the rate per unit area that energy is transported via an electromagnetic wave using the Poynting vector.
- Perform calculations to relate the intensity of an electromagnetic wave to the time average rate at which it transports energy and the magnitude of the electric field, the power supplied by a source and its distance from the source.
- Calculate the force applied on a surface by an electromagnetic wave which is either totally absorbed by the surface or totally reflected from a surface. Calculate the radiation pressure for both these cases.

Maxwell's Equations—integral form

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{enc}}}{\epsilon_0}$$

$$\oint \vec{B} \cdot d\vec{A} = 0$$

$$\oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt}$$

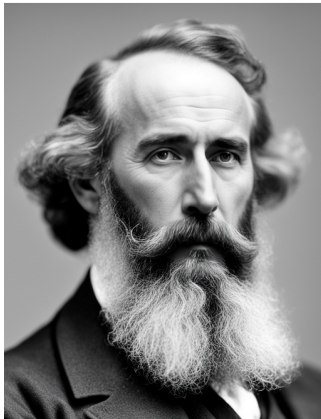
$$\oint \vec{B} \cdot d\vec{s} = \mu_0 \epsilon_0 \frac{d\Phi_E}{dt} + \mu_0 i_{\text{enc}}$$

- These are the equations you have been learning about in Part 1 of this course.
- You should know what each symbol means.
- You should know what each term means.
- You should know what each equation means.

Mini-quiz on Maxwell's equations



Maxwell's Equations of Electromagnetism



Which of the following involve line integrals?

- ☐ $\oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\epsilon_0}$
- ☐ $\oint \vec{B} \cdot d\vec{A} = 0$
- ☐ $\oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt}$

Maxwell's Eqns—differential form

$$\nabla \cdot \vec{E} = \frac{\rho}{\epsilon_0}$$

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \times \vec{B} = \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t} + \mu_0 \vec{J}$$

- These are the same equations, in an alternative form.
- You will use this form in Level 2/3 courses.
- The del operator ∇ is defined by $\nabla = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right)$
- ∇ is also called “grad”, $\nabla \cdot$ is called “div” and $\nabla \times$ is called “curl”.
- You should aspire to having an intuitive understanding of what these mean.

Maxwell's Equations—differential form in a vacuum

$$\nabla \cdot \vec{E} = 0$$

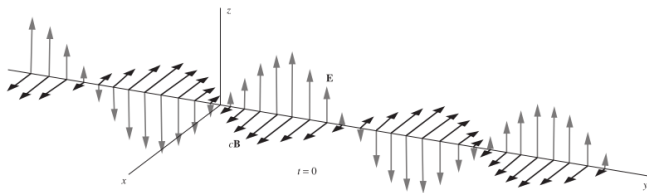
$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \times \vec{B} = \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$$

These equations are non-examinable

- In a vacuum, ρ and $\vec{J}$ are both zero, so we can simplify the equations from the last slide.
- The equations are now very symmetrical.
- One solution is: $\vec{E} = \vec{z}E_{\max} \cos(ky - \omega t)$ and $\vec{B} = \vec{x}B_{\max} \cos(ky - \omega t)$, which you should recognise as a traveling wave.
- The wave velocity is $v = \pm \frac{1}{\sqrt{\mu_0 \epsilon_0}} = \pm c$, and $E_{\max} = \pm cB_{\max}$.
- Maxwell calculated v from laboratory measurements of E and B , and was amazed to discover it was identical to the speed of light!



■ This shows a plane EM wave moving in the $+y$ direction.

■ The $\vec{E}$ and $\vec{B}$ vectors are independent of x and z .

NOTE:

1. The wave travels with speed c .
2. At every point in the wave, at any instant of time, $E = cB$.
3. $\vec{E}$ and $\vec{B}$ are perpendicular to one another, and to the direction of propagation.

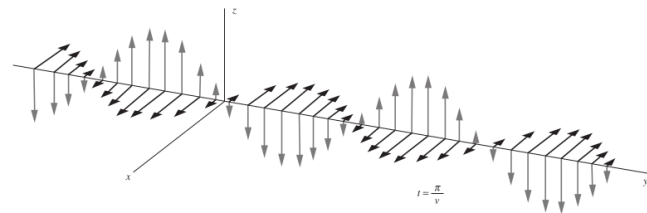
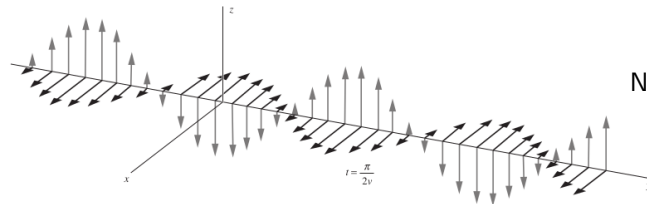
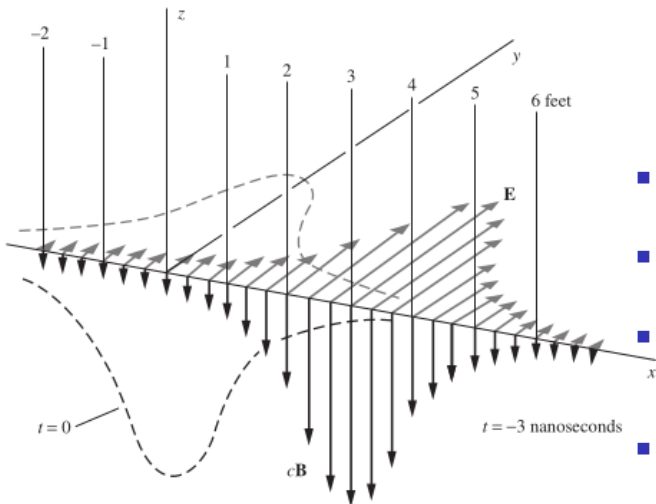


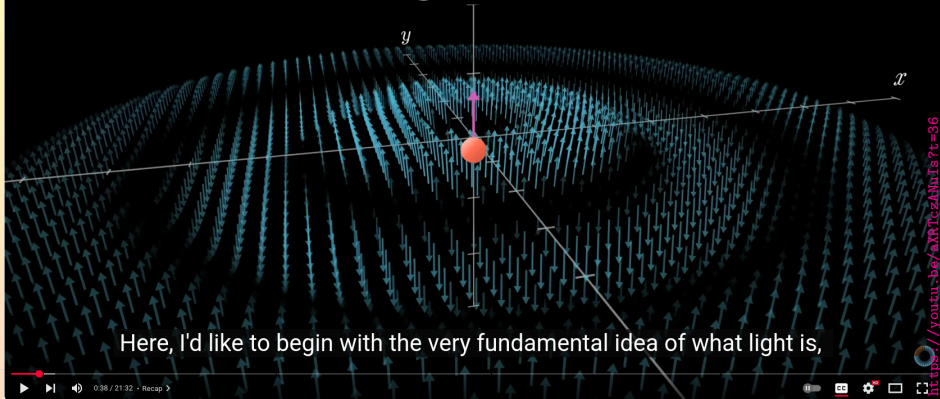
Figure 9.7.

The wave described by Eqs. (9.22) and (9.23) is shown at three different times. It is traveling to the right, in the positive y direction.



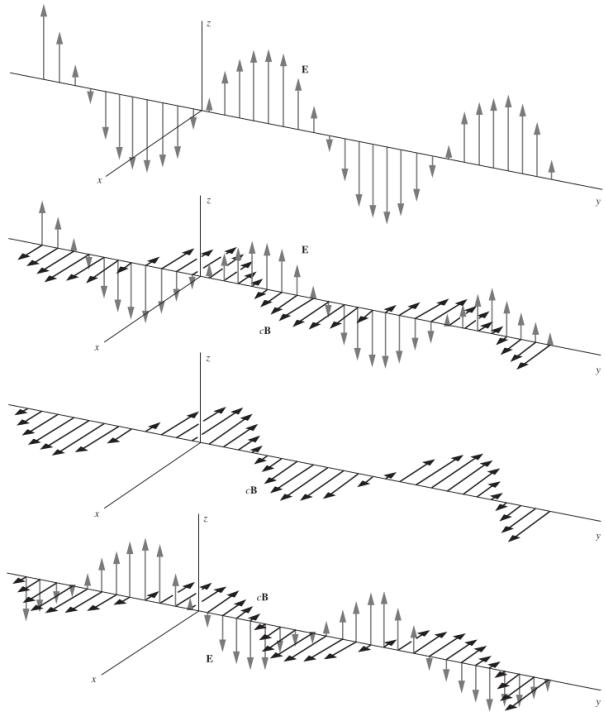
- EM waves do not have to be sinusoidal.
- They can be any function you like.
- The wave on the left is a pulse propagating in the $+x$ direction.
- The wave satisfies the three conditions from the previous slide.

Electromagnetic Radiation



3Blue1Brown: How wiggling charges give rise to light (the link to video is on the right edge)

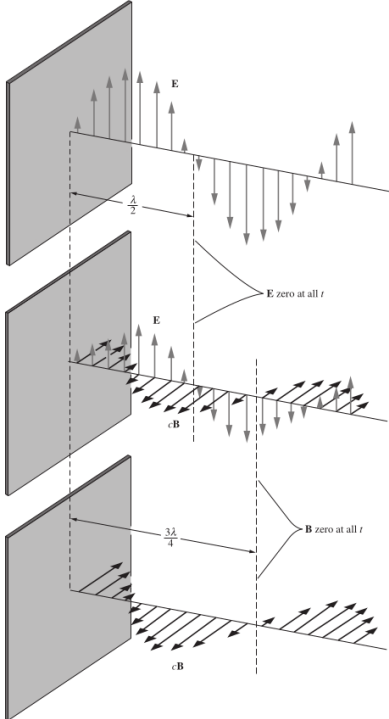
The video contains much more information than is examinable, but is helpful to understand how EM waves propagate. We will come back to this video later when we study polarisation.



Just like waves on a string, you can have two EM waves traveling in opposite directions adding to produce a standing wave, with nodes and anti-nodes.

NOTE:

1. The wave is stationary.
2. E does not necessarily equal cB .
3. $\vec{E}$ and $\vec{B}$ are perpendicular to one another.



- This is just the previous slide's standing wave, with a conducting sheet placed at a node where $E = 0$.
- You can think of this as an EM wave traveling at speed c producing a standing wave upon reflection at a perfectly conducting sheet.
- We can place additional sheets at other E nodes, and hence trap a wave between two mirrors. This is used in a laser, and also a microwave oven.

Figure 9.10.

A standing wave produced by reflection at a perfectly conducting sheet.

Physics of the microwave oven

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Abstract

This is the first of two articles about the physics of microwave ovens. This article deals with the generation of microwaves in the oven and includes the operation of the magnetrons, waveguides and standing waves in resonant cavities. It then considers the absorption of microwaves by foods, discussing the dielectric relaxation of water, penetration depths of electromagnetic waves in matter and, in considering the possible chemical changes during the microwave heating, multi-photon ionization or dissociation.

What are microwaves?

Microwaves are electromagnetic waves. Their frequencies (wavelengths) are in the range from 300 MHz ($\lambda = 1$ m) up to 300 GHz ($\lambda = 1$ mm). Regarding wavelengths as typical spatial dimensions, one realizes that microwaves do not have dimensions of μm , as might be expected from the misleading 'micro' in their name. Following international conventions, microwave ovens at home or in restaurants operate at frequencies of about 2.45 GHz, i.e. $\lambda = 12.23$ cm.

Physics of a microwave oven

Figure 1 depicts a typical microwave oven (many details can be found in [1–3]). Microwaves are generated in a magnetron which feeds via a waveguide into the cooking chamber. This cuboid chamber has metallic walls and so acts as a Faraday cage. The front door, made of glass, and the light bulb cavity are both covered by metal grids. The holes in the grids are small compared with the wavelength of the microwaves, hence the grids act just like metal plates.

Most microwaves cook the food on a rotating turntable in this chamber, but some designs include a rotating reflector, acting as a stirrer. Expensive models may include thermometers, additional

1. Microwave oven

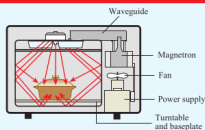


Figure 1. Schematic diagram of a typical microwave oven.

conventional cooking facilities such as grills, oven heaters and even refrigeration.

Interaction of microwaves with metals

Microwaves, incident on the metal walls of the oven, behave similarly to visible light hitting a silver mirror (e.g. [4]). The microwaves are absorbed very effectively, since the electric fields of the waves interact very strongly with the nearly free electrons of the metal. In a simple model, the electron behaviour is described as a damped forced oscillation. These accelerated electrons re-radiate electromagnetic waves at the same frequency and

in phase, hence the microwaves are perfectly reflected. Macroscopically, this behaviour is described by the complex dielectric constant $\epsilon(\omega)$, which is the square of the complex refractive index ($\epsilon_1 + i\epsilon_2 = (n_1 + in_2)^2$).

The refractive index of many metals gives reflectivities close to 100% at low frequencies. The penetration depth of electromagnetic waves of wavelength λ is given by

$$\delta = \lambda / 4\pi n_2. \quad (1)$$

For example, for microwaves with $\lambda = 12.2$ cm incident on aluminium, $\delta \approx 1.2 \mu\text{m}$.

These are similar to skin depths, i.e. the attenuation depths of alternating currents of frequency ω in metals. (The relation between skin depth and refractive index for small frequencies is discussed, e.g., by Feynman [5].)

Generating microwaves in magnetrons

The most powerful microwaves produced by solid state devices, such as used in cell phones, are far too weak for cooking. Instead electron beams in vacuum tubes under the combined effect of electric and magnetic fields are made to follow curved trajectories (the detailed mechanism for which is described below). Most microwave ovens use magnetrons. First invented in 1921 and strongly improved around 1940, magnetrons allow either continuous or pulsed microwave generation with powers up to megawatts and frequencies between 1 and 40 GHz. Efficiencies are around 80% and lifetimes about 5000 hours.

A cylindrical cathode is at the axis, several millimetres from a hollow circular anode (figure 2). Inside the anode there are a number of cavities designed to resonate at 2.45 GHz. A voltage of several kV is applied between the electrodes and a magnetic field is applied parallel to the axis such that electric and magnetic fields are perpendicular to each other.

Electrons ejected by the cathode accelerate radially at first, but because of the magnetic field they start to follow cycloidal paths. If the magnetic field is strong enough, the electrons cannot reach the anode but form a rotating space charge. The resonant cavities of the anode interact with the electrons by either accelerating or decelerating them. Finally this leads to electron bunches which move around the cathode at microwave

2. The magnetron

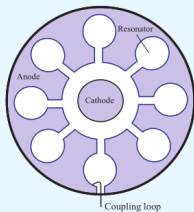


Figure 2. Schematic diagram of a magnetron.

frequencies, which in turn leads to self-sustaining oscillations of the resonant cavities. Part of the microwave power is extracted by a coupling loop (for more details, see [1–3]).

The magnetrons in domestic microwave ovens emit microwaves at 2.45 GHz (repeatable, each time the magnetron is switched on, to ± 10 MHz) with bandwidths of only a few MHz [6].

Connecting the magnetron with the cooking chamber: the waveguide

Waveguides carry the microwaves from the magnetron to the cooking chamber. They are usually metal tubes of rectangular cross section. The effect of the boundary conditions of the tubes leads to electric and magnetic field distributions which, in each of the two directions perpendicular to the axis, resemble standing waves [1, 5]. These are quite similar to standing waves on the 'one-dimensional' string of a guitar. Consequently, there is a maximum wavelength, λ_{max} , which can be transported by the waveguide. For a given direction perpendicular to the axis, the inner tube size is then just $\lambda_{\text{max}}/2$. Therefore, a waveguide filled with air which operates at 2.45 GHz ($\lambda = 12.2$ cm) must have at least one inner dimension larger than 6.1 cm. Usually, the waveguide has a smaller size in a perpendicular direction. Filling

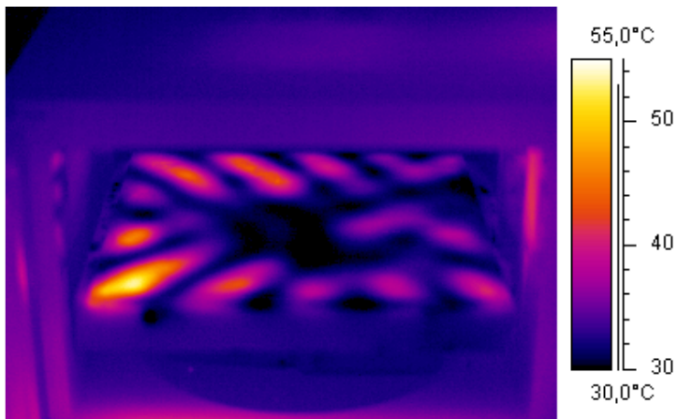
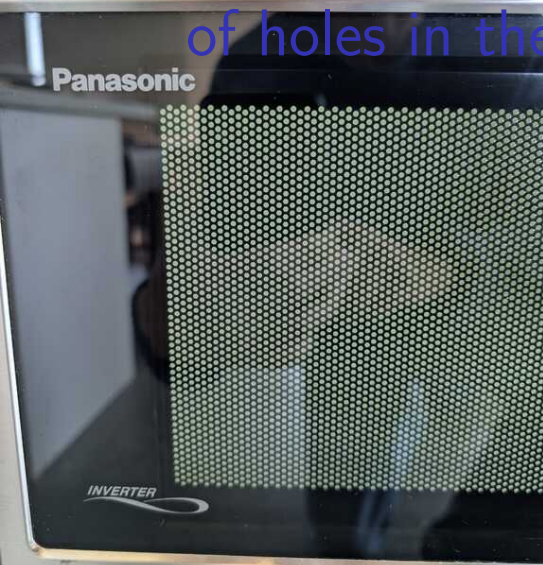


Figure 4. Visualization of the horizontal mode structure in a microwave oven using infrared thermal imaging. A glass plate with a thin water film was placed at a height of 8 cm and heated for 15 s with a microwave power of 800 W without using the turntable (for more details on the experiment, see [10]).

Why do microwave ovens have lots of holes in their doors?



Demo—white light and a prism



Energy transport—the Poynting vector

- The Poynting vector is defined as,

$$\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B}$$

- Its magnitude, S , gives the instantaneous power per unit area transported by the EM wave. Its direction is the same as the propagation direction of the wave.
- Since in an EM wave $\vec{E}$ and $\vec{B}$ are perpendicular, and $E = cB$,

$$S = \frac{1}{c\mu_0} E^2$$

- We are usually more interested in the average value of S over a cycle, which we call the intensity I , which is

$$I = \frac{1}{c\mu_0} E_{\text{rms}}^2$$

where E_{rms} is the square root of the average value of E^2 over a cycle.

Intensity as a function of distance

- We are often interested in how the intensity of a wave varies with distance from the source.
- If the source of the EM waves is a point radiating isotropically with power P_s , then the intensity (power per unit area) at a radius r from the point is just

$$I = \frac{P_s}{4\pi r^2}$$

- If the source is an infinite line, the intensity drops off as $\frac{1}{r}$.
- If the source is an infinite plane, the intensity is independent of distance.

Radiation pressure

- EM waves exert a force (and hence a pressure) on surfaces they encounter.
- If the EM radiation is totally absorbed by the surface, the force and pressure are

$$F = \frac{IA}{c} \qquad p_r = \frac{I}{c}$$

where A is the area perpendicular to the propagation direction.

- If the EM radiation is totally reflected, the momentum change is doubled, and the force and pressure are

$$F = \frac{2IA}{c} \qquad p_r = \frac{2I}{c}$$

Mini-quiz on radiation pressure



Radiation pressure on a comet



A comet is heated by the Sun and begins to evaporate, producing gas (ions) and dust (macroscopic particle). The gas tail is due to magnetic fields from charged wind coming from the Sun. The dust tail is due to radiation pressure, and is pushed out much more slowly.

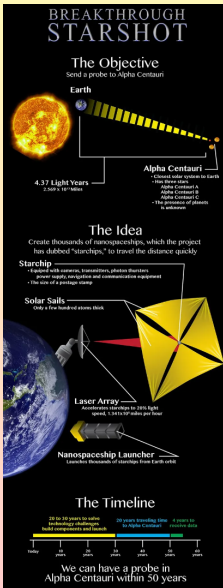
In the image, is the ion tail blue or white?

- ☐ Blue
- ☐ White

In which direction is the Sun?

Project Breakthrough Starshot

- A StarChip is a minimal spacecraft with a mass of a few grams. A baseline design has five 2 Mpixel digital cameras, 4 microprocessors, 4 photon thrusters, and a 150 mg atomic battery.
- StarChips are accelerated at 10,000 g for 10 minutes to 0.20 c, using radiation pressure from a 100 GW square-km phased-array of lasers.
- The proposal is for 10 m² solar sails.
- 1000 StarChips will be sent to allow for failures from collisions, etc., along the way.
- The travel time to Alpha Centauri is ~20 years.
- In 2025 a new solar sail design was developed (see next slide).
- The project was put on hold in September 2025.





Pentagonal photonic crystal mirrors: scalable lightsails with enhanced acceleration via neural topology optimization

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Check for updates

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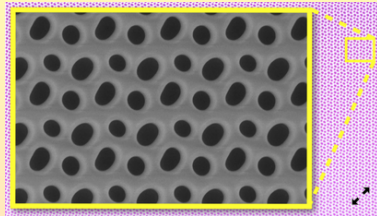
The Starshot Breakthrough Initiative aims to send gram-scale microchip probes to Alpha Centauri within 20 years, propelled by laser-driven lightsails at a fifth of light speed. This mission demands innovative lightsail materials with meter-scale dimensions, nanoscale thickness, and billions of nanoscale holes for enhanced reflectivity and reduced mass. Unlike the microchip payload, lightsail fabrication requires breakthroughs in optics, materials science, and structural engineering. Our study uses neural topology optimization, revealing a novel pentagonal lattice-based photonic crystal (PhC) reflector. The optimized designs significantly lower the acceleration times and, thereby, launch cost. Crucially, they also enabled orders-of-magnitude fabrication cost reduction. We fabricated a $60 \times 60 \text{ mm}^2$, 200 nm thick reflector with over a billion nanoscale features, achieving a 9000-fold cost reduction per m^2 . This represents the highest aspect ratio nanophotonic element to date. While stringent requirements remain for lightsails, scalable, cost-effective nanophotonics present promising solutions for next-generation space exploration.

Currently, the human-made object furthest from Earth is the Voyager 1. Traversing space since 1977, this spacecraft has only recently left our solar system, a mere 0.5% of the distance to the nearest star outside our solar system: Alpha Centauri. With existing propulsion systems, approaching our nearest interstellar neighbor would take over 10,000 years. In 2016, the Breakthrough Prize Foundation announced the Starshot Initiative to push the development of low-mass microchip satellites with cameras, sensors, and probes accelerated to high speeds by low-mass lightsails¹. The Starshot Mission leverages advances in nanotechnology to create low-mass objects, and achieve progress in high-power lasers that beam energy to distant locations as far as tens of millions of kilometers away².

This microchip approach to space exploration aims to reach Alpha Centauri (i.e., the nearest star outside our Solar system) within 20 years by reaching speeds up to 20% of the speed of light, made possible by the rapidly advancing field of nanotechnology and the future potential of next-generation laser systems. Regardless of the payload mass, this mission is fundamentally exploring the physical limits of mass acceleration and our ability to reach relativistic speeds with novel materials made possible by nanotechnology. Of the many ambitious developments required by the Starshot Initiative, the lightsails are generally considered one of the most challenging components to realize due to their unique geometries and stringent performance requirements.

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The Starshot solar sail



- The sail needs to have high reflectivity over a fairly wide wavelength range, due to the Doppler shift of the laser light, which has a rest wavelength of $1.06 \mu\text{m}$.
- The sail also has to be very thin, to keep its mass down to ~ 1 gram.
- The new design uses a 200 nm thick SiN film, with a series of sub-wavelength 500 nm “potato-shaped” holes in a hexagonal pattern. This is an example of a “photonic crystal”. Perhaps surprisingly, this leads to a much higher reflectivity than a surface with no holes.
- The high reflectivity can be understood by diffraction of light from the holes, with the sub-wavelength size only allowing the zero-order (i.e., reflection).

The *Pioneer* Anomaly



- The *Pioneer 10* and *Pioneer 11* spacecraft were launched in the early 1970s to study the outer solar system.
- Detailed study of the navigation data in the 1990s showed that the spacecraft were slowing slightly more than expected. The anomalous acceleration was $-8 \times 10^{-10} \text{ m/s}^2$.
- For many years it was speculated that this could even be an indication of a problem with the inverse square law of gravity.
- The problem wasn't resolved until 2012, when radiation pressure was found to be the cause.

Support for the Thermal Origin of the Pioneer Anomaly

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We investigate the possibility that the anomalous acceleration of the Pioneer 10 and 11 spacecraft is due to the recoil force associated with an anisotropic emission of thermal radiation off the vehicles. To this end, relying on the project and spacecraft design documentation, we constructed a comprehensive finite-element thermal model of the two spacecraft. Then, we numerically solve thermal conduction and radiation equations using the actual flight telemetry as boundary conditions. We use the results of this model to evaluate the effect of the thermal recoil force on the Pioneer 10 spacecraft at various heliocentric distances. We found that the magnitude, temporal behavior, and direction of the resulting thermal acceleration are all similar to the properties of the observed anomaly. As a novel element of our investigation, we develop a parametrized model for the thermal recoil force and estimate the coefficients of this model independently from navigational Doppler data. We find no statistically significant difference between the two estimates and conclude that, once the thermal recoil force is properly accounted for, no anomalous acceleration remains.

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PACS numbers: 04.80.-y, 95.10.Eg, 95.55.Pe

Introduction.—The anomalous acceleration of the Pioneer 10 and 11 spacecraft [1] is a discrepancy between modeled and observed radiometric Doppler data received from the two vehicles. The discrepancy can be eliminated by incorporating a constant sunward acceleration of unknown origin with a magnitude of $a_P = (8.74 \pm 1.33) \times 10^{-10} \text{ m/s}^2$ into the orbital model [2]. The presence of this acceleration, which became known as the Pioneer anomaly, was seen by many as a breakdown in the gravitational inverse-square law of general relativity that reveals itself in the dynamics of the outer solar system. There were also proposals that this anomaly was due, at least in part, to the waste heat emitted by the spacecrafts' radioisotope thermoelectric generators (RTGs) [3], their electrical subsystems [4], or both [5], motivating a thorough investigation of the spacecraft systematics (for a review, see [6]).

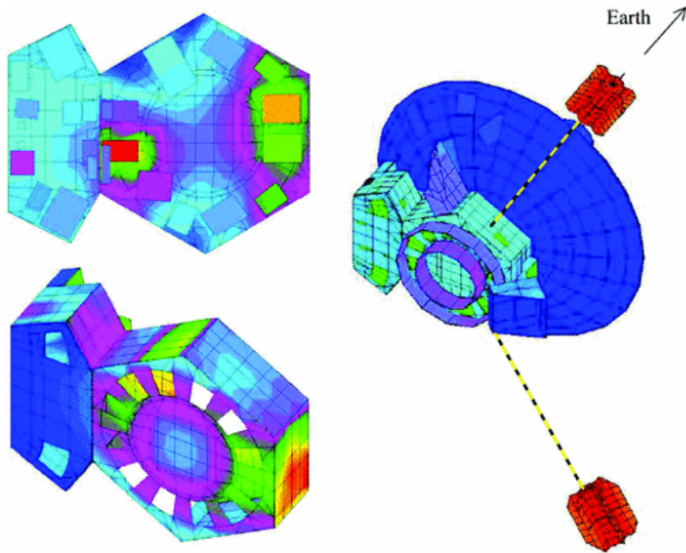
Our current investigation began with the recovery of the entire telemetry record of both spacecraft, substantial project documentation, and additional Doppler data [7]. The analysis of the extended Doppler data set was completed recently [8], confirming the existence of the anomaly. This analysis also showed that, although the direction of the anomalous acceleration cannot be determined unambiguously, the Doppler residuals improve if one considers a temporally varying acceleration in the direction of the Earth, which is consistent with earlier, similar results [9,10]. These results are suggestive: If the acceleration were due to thermal recoil force, it would be along the spacecraft spin axis, which generally points in the direction of the Earth and would have a temporally decreasing magnitude consistent with the decay of the on-board radioisotope fuel (^{238}Pu).

In this Letter, we report on the completion of a finite-element thermal model of the Pioneer 10 spacecraft. This model relies extensively on the project and spacecraft design documentation and was validated using redundant flight telemetry data. A parametrized form of the thermal force model [11] was also incorporated into a Doppler analysis, which yielded an independent estimate of coefficients characterizing the thermal recoil force. We compare the outputs of the two independent analyses—Doppler and thermal—to show that no statistically significant anomalous acceleration remains in the data.

Finite-element thermal model.—A comprehensive finite-element (FE) thermal model (Fig. 1) of the Pioneer 10 and 11 spacecraft was constructed at the Jet Propulsion Laboratory (JPL) in collaboration with the Applied Sciences Laboratory. The geometric and thermal models of the spacecraft were constructed using the SINDA-3D thermal modeling software [12]. While the software provides the capability to build a numerical model directly from computeraided design (CAD) drawing files, no such files exist for a spacecraft designed 40 years ago. For this reason, the model was built in a more tedious manner by specifying the coordinates of the vertices of each modeled spacecraft surface, using available blueprints and recovered project documentation. The spacecraft geometric model was built with a Monte Carlo based radiation analyzer [Thermal Synthesizer System (TSS)] to calculate the radiative exchange factors using infrared emittance values for modeled surfaces specified within it. The model incorporated approximately 3300 surface elements, 3700 nodes, and 8700 linear conductors. The spacecraft thermal-mechanical configuration is

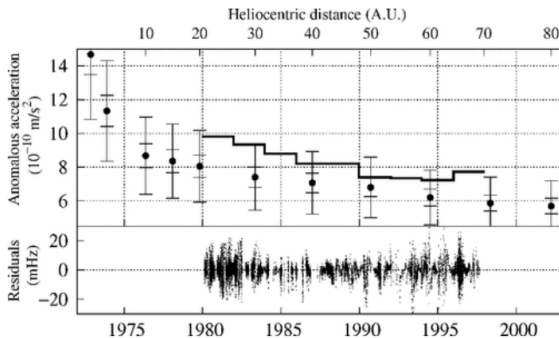
- The anomalous acceleration was detected by a precise analysis of the Doppler shift of the radio communication from the spacecraft.
- The anomaly wasn't simply due to mistakes accounting for radiation pressure from the Sun.
- The issue was thermal (infrared) EM radiation from the spacecraft itself, requiring a detailed analysis of the temperature of all surfaces of the spacecraft, including the Radioisotope Thermal Generators (RTGs), and of the path of the radiation being emitted.

FIG. 1.



Illustrative representation of the thermal model of the Pioneer 10 spacecraft evaluated at 40 A.U. Top left: spacecraft interior (temperature range: blue -16°C , red $+10^{\circ}\text{C}$); bottom left: spacecraft exterior (blue -155°C , red -108°C); right: end view (temperature range: blue -213°C , red $+136^{\circ}\text{C}$). Unmodeled struts that connect the RTGs to the spacecraft body are indicated with yellow and black dashed lines.

FIG. 3.



Comparison of the thermally induced and anomalous accelerations for Pioneer 10. The estimated thermal acceleration is shown with error bars. The stochastic acceleration estimate from [8] appears as a step function. For reference, the Doppler residuals of the stochastic acceleration are also shown in the bottom panel. Inner and outer error bars correspond to the subtotal and total shown in Table II.

Special Relativity

- Einstein's Theory of Special Relativity is a direct result of the speed of light, c , being the maximum allowed speed.
- A consequence is that velocities don't simply add. Instead

$$u = \frac{v + u'}{1 + (vu'/c^2)}$$

- Also, mass is not invariant with velocity (objects become more massive as they go faster; if they didn't, you would be able to make them go faster than c by applying a constant force). Fortunately, charge is invariant (this is very important to electromagnetism!).
- Objects traveling at a speed $v = \beta c$ experience length contraction along the direction of motion by the Lorentz factor $\gamma = \frac{1}{\sqrt{1-\beta^2}}$.
- Time slows down by the same factor.
- Another consequence is that the E field from a moving charge is no longer isotropic, but is strongest perpendicular to the direction of motion, as shown in the next two slides.

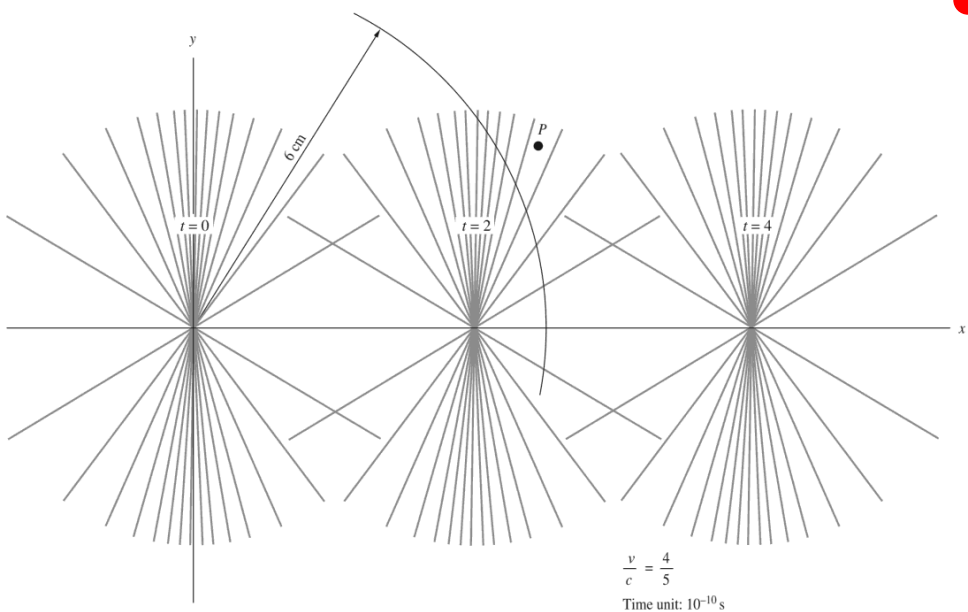


Figure 5.17.
The electric field of a moving charge, shown for three instants of time; $v/c = 4/5$.

have received the news that the electron had started to move at $t = 0$!

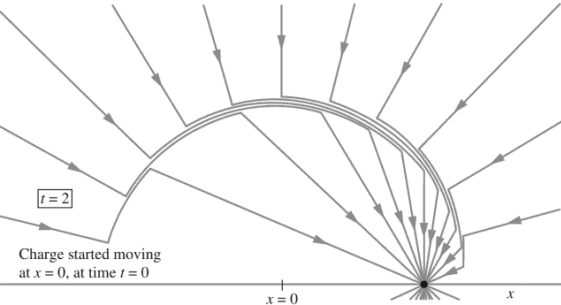


Figure 5.18.

An electron initially at rest in the laboratory frame is suddenly accelerated at $t = 0$ and moves with constant velocity thereafter. This is how the electric field looks at the instant $t = 2$ all over the laboratory frame.

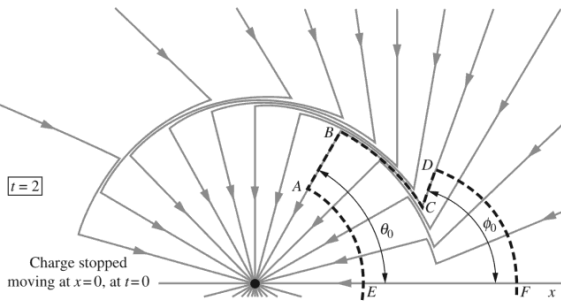
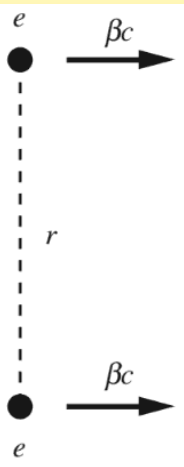


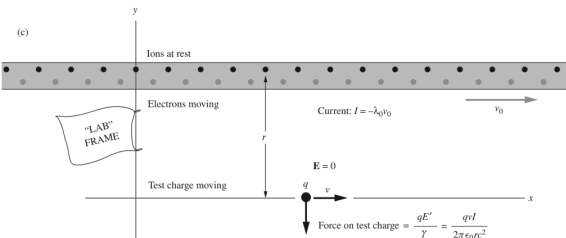
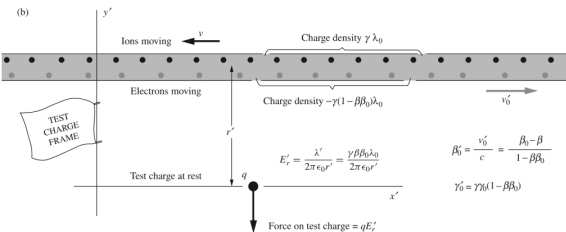
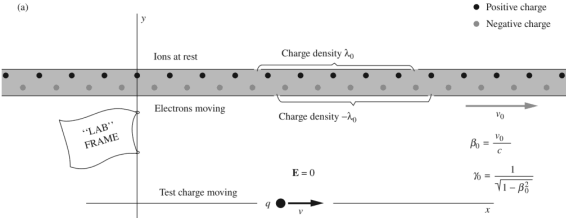
Figure 5.19.

An electron that has been moving with constant velocity reaches the origin at $t = 0$, is abruptly stopped, and remains at rest thereafter. This is how the field looks in the laboratory frame at the instant $t = 2$. The dashed outline follows a field line from A to D. Rotating the whole outline $EABCD$ around the x axis generates a closed surface, the total flux through which must be zero. The flux in through the spherical cap FD must equal the flux out through the spherical cap EA . This condition suffices to determine the relation between θ_0 and ϕ_0 .

Maxwell's eqns & inertial frames



- Consider two protons, with charge $+e$, traveling to the right at velocity βc .
- In the rest frame, the upper proton produces a magnetic *and* electric field at the position of the lower proton.
- Now change to the frame moving with the protons.
- In this frame the upper proton only produces an electric field at the position of the lower proton!
- How do we reconcile these two approaches?
- The amazing answer is that we need to use Einstein's Theory of Special Relativity.
- It turns out that *what we think of as magnetism is simply a combination of Coulomb's Law and Special Relativity.*



In the lab frame, the ions are stationary and the electrons (after length contraction) are spaced the same as the ions, making the wire neutral. The test charge experiences a magnetic force from the electrons in the wire (as shown in (c)).

In the test charge frame, the ions in the wire are moving to the left, and the electrons to the right. Since the speeds are different, they experience different amounts of length contraction, so the wire appears to be charged! The test charge experiences an electric force from this charge, but no magnetic force (since the charge is stationary in this frame).

(c) shows how to calculate the magnetic force in the lab frame from the electric force in the moving frame. The bottom line is that magnetism is an effect due to Special Relativity.