

Lecture 11—learning outcomes

- Define the terms “coherent”, “monochromatic” and “diffraction”.
- Recall from Physics 1A the conditions under which waves interfere constructively and destructively.
- Draw a diagram showing how to set up Young’s double slit experiment. For this setup calculate where maximum and minimum intensity fringes will be observed.
- Use phasors to calculate the intensity when coherent waves meet.
- Predict the intensity of light on the viewing screen in Young’s double slit experiment.
- Predict intensities when more than two coherent waves combine using phasors.

Textbook chapters: 35.2–3 <https://ebookcentral.proquest.com/lib/unsw/reader.action?c=UERG&docID=30765671&ppg=853>

Diffraction from a slit

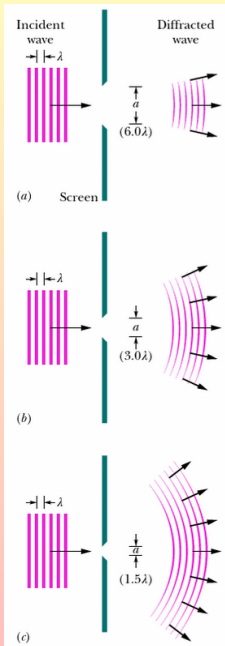
The figure shows diffraction as a function of slit width (measured in units of the wavelength λ of the light).

The slit extends well into and out of the page, and perpendicular to it.

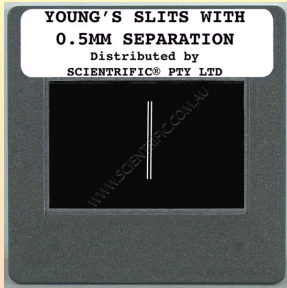
We will cover the details of single slit diffraction in a later lecture, but for now all you need to know is that for a given λ , the diffraction is more pronounced the smaller the slit width a ; similarly, for a given slit width, the diffraction is more pronounced for longer wavelengths..

The figures show the cases for a slit width of

- (a) $a = 6.0\lambda$,
- (b) $a = 3.0\lambda$, and
- (c) $a = 1.5\lambda$.



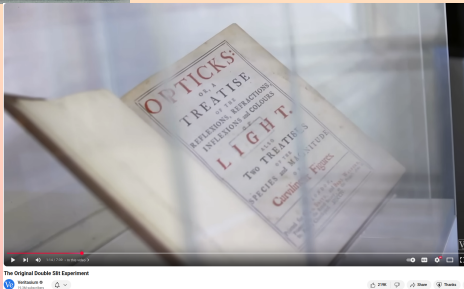
What would you expect to see with a *double* slit?



The figure shows two narrow slits, 0.5mm apart, in a thin sheet of metal.

If you passed sunlight through the slits, what would you see on a screen a metre away on the other side?

This experiment was performed by Thomas Young in 1801, and it proved that light acts as a wave, contrary to the widely held view at the time that light consists of "corpuscles" (particles).



<https://www.youtube.com/watch?v=Iuv6hY6zsd0&t=59>

Demo: Young's double slit experiment—under construction yesterday



Young's Experiment

Sunlight

Narrow slits

Observing screen

Pattern observed on screen

Right slit open

observed

Left slit open

observed

Both slits open

wave theory

particle theory

observed

Possible outcomes of Young's experiment versus observations

Sunlight diffracted through a double slit

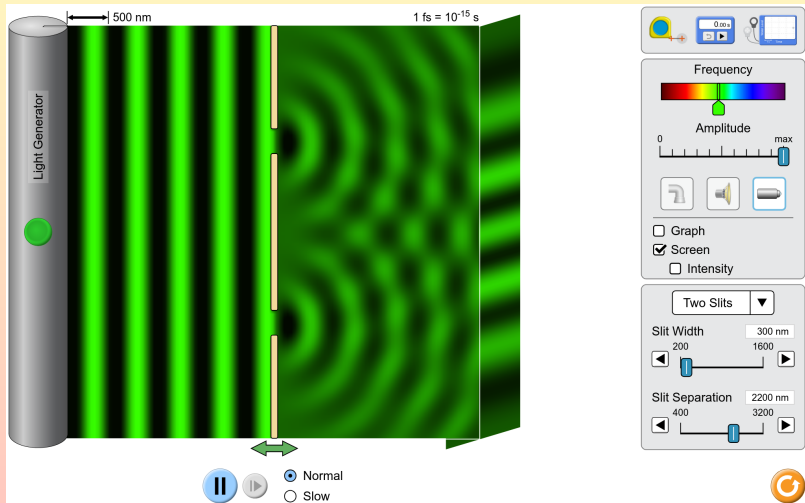


The figure shows sunlight hitting a screen after diffraction through a double slit.

Each wavelength present in the sunlight is defracted by a different amount, producing fringes separated by different distances. Blue light by shorter distances, red by longer.

The combined effect is shown in the figure.

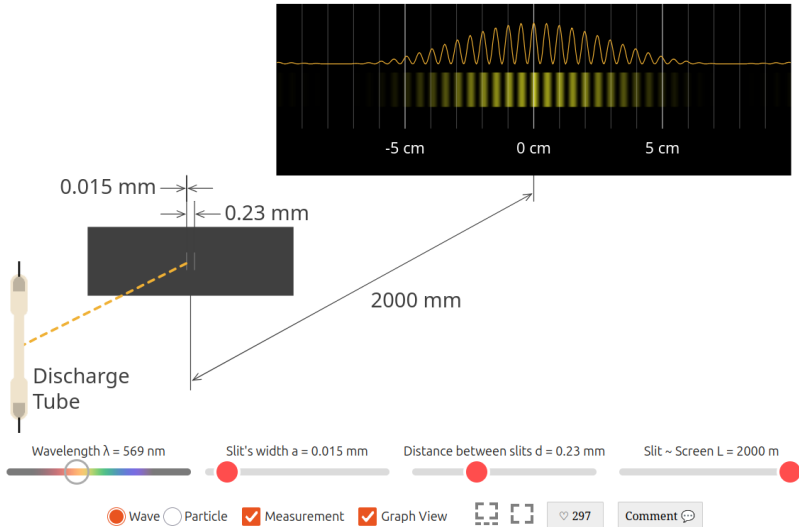
Double slit experiment simulation 1



<https://phet.colorado.edu/sims/html/wave-interference/latest/wave-interference>

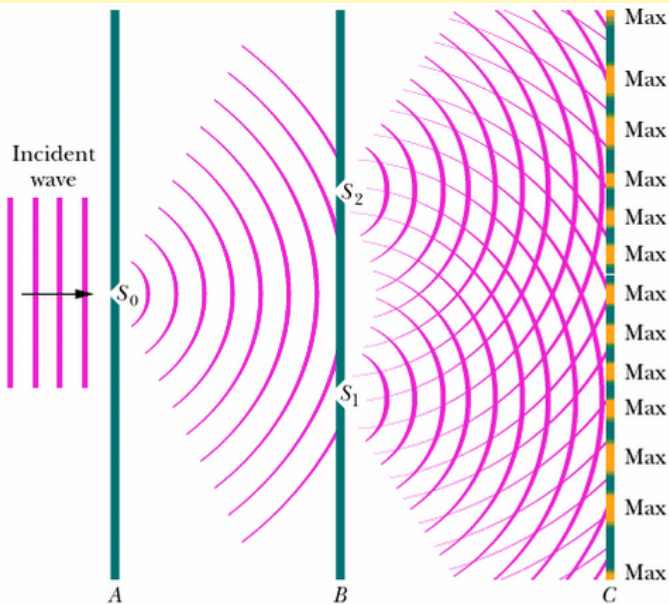
Double slit experiment simulation 2

Double-Slit Experiment



https://javalab.org/en/double_slit_en/

Young's double slit experiment



The waves emerging from the two slits overlap and form an interference pattern.

What is the purpose of the single slit in the previous slide?

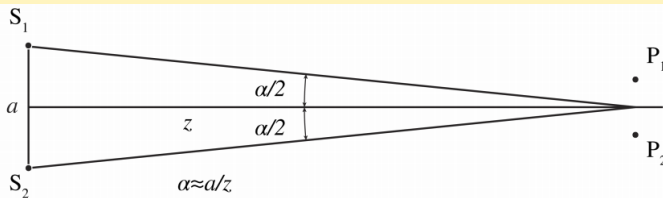
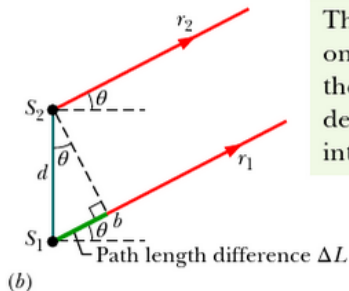
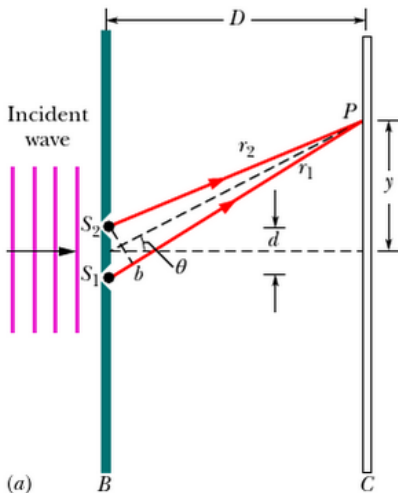


Figure 5.8.1: Two incoherent point sources S_1, S_2 and two points P_1, P_2 in a plane at large distance z from the point sources. The degree of mutual coherence at P_1 and P_2 increases to 1 in the limit $\alpha \rightarrow 0$.

- Sunlight is not a coherent light source like a laser, so how can we use it for Young's double slit experiment?
- Well, sunlight is *partially coherent*, i.e., it is coherent over sufficiently short times, and between sufficiently closely placed slits.
- The figure shows two points S_1 and S_2 on the Sun, the two observers P_1 and P_2 on the Earth. Let, e.g., $P_1 S_1$ be the distance from P_1 to S_1 , then provided that $abs((P_1 S_1 - P_1 S_2) - (P_2 S_1 - P_2 S_2)) < \lambda/2$ then the light at P_1 and P_2 will be coherent enough to show interference.
- This gives the result that $P_1 P_2 < 2\alpha\lambda$, where α is the angle subtended by the Sun from Earth (about 0.5°).
- The distance $P_1 P_2$ is about $30\mu\text{m}$ for sunlight, so our slits need to be closer than this to see clear fringes. In Veritasium's video, the slit separation was about $200\mu\text{m}$ —I don't know how it was able to work (see the next lecture for an explanation!).
- Young's solution was to first pass the sunlight through a narrow single slit, which then produced coherent light for the more widely spaced double slits.

Path length difference from two slits

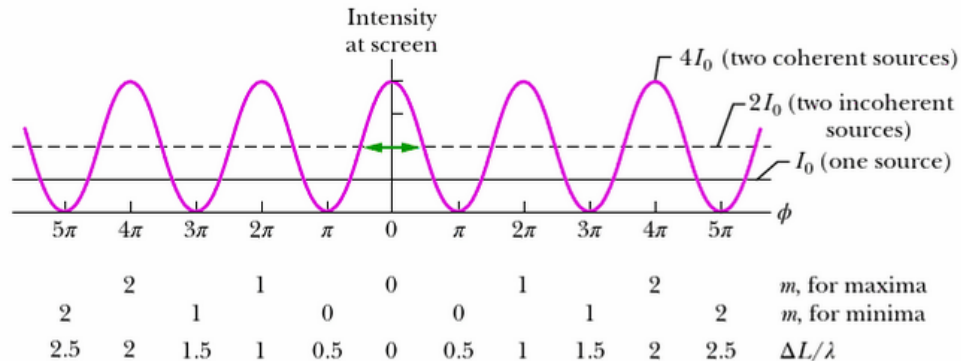


The ΔL shifts one wave from the other, which determines the interference.

Demo: Lb09—Young's double slit experiment



Intensity in double-slit interference



The figure shows the intensity of a double slit interference pattern as a function of the phase differences between the waves when they arrive from the two slits. I_0 is the uniform intensity that would appear on the screen if one slit were covered. The average intensity of the fringe pattern is $2I_0$, and the maximum intensity for (coherent light) is $4I_0$.

$$I = 4I_0 \cos^2 \frac{1}{2}\phi$$

What is a “phasor”?

Hopefully you recall what phasors are from previous studies of waves. If not, here is a quick refresher, as applied to the electric field at a point.

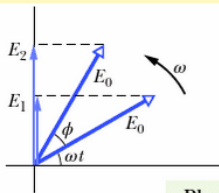
- If the electric field at a point is varying sinusoidally with an angular frequency ω , it can be represented with a function as follows:

$$E(t) = E_0 \sin(\omega t + \phi)$$

where E_0 is the amplitude, and ϕ is the phase constant.

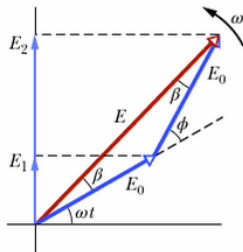
- You can represent the above equation as a “phasor”, which is a 2D vector with length E_0 , and an angle $\omega t + \phi$ measured CCW from the $+x$ -axis. $E(t)$ is then the y component of the vector as it rotates CCW with increasing time.
- If the electric field consists of several sinusoidal components, you can simply add the phasors vectorially to determine the resultant.
- In the special case that the components have the same ω , but possibly different amplitudes and phase constants, the phasors rotate together in a fixed pattern, which makes it particularly easy to solve.

Combining light waves using phasors



(a)

Phasors that represent waves can be added to find the net wave.

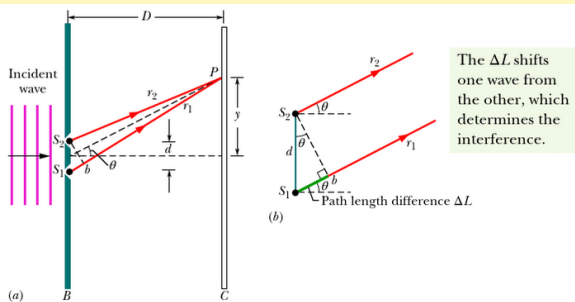


(b)

The top figure shows phasors representing, at time t , the electric field components of two light waves (one from each slit in the double-slit experiment). Both phasors have magnitude E_0 and rotate counterclockwise with angular speed ω . Their phase difference is ϕ .

The bottom figure shows the vector addition of the two phasors giving the phasor representing the resulting wave with amplitude E and phase $\omega t + \beta$.

Problem—double slit interference



What is the distance on the screen between adjacent maxima near the centre of the interference pattern?

Assume the wavelength of the light λ is 546 nm, the slit separation d is 0.12 mm, and the slit-to-screen separation D is 55 cm.

Solution

Sample Problem 35.2.1 Double-slit interference pattern

What is the distance on screen C in Fig. 35.2.5a between adjacent maxima near the center of the interference pattern? The wavelength λ of the light is 546 nm, the slit separation d is 0.12 mm, and the slit-screen separation D is 55 cm. Assume that θ in Fig. 35.2.5 is small enough to permit use of the approximations $\sin \theta \approx \tan \theta \approx \theta$, in which θ is expressed in radian measure.

KEY IDEAS

(1) First, let us pick a maximum with a low value of m to ensure that it is near the center of the pattern. Then, from the geometry of Fig. 35.2.5a, the maximum's vertical distance y_m from the center of the pattern is related to its angle θ from the central axis by

$$\tan \theta \approx \theta = \frac{y_m}{D}.$$

(2) From Eq. 35.2.3, this angle θ for the m th maximum is given by

$$\sin \theta \approx \theta = \frac{m\lambda}{d}.$$

Calculations: If we equate our two expressions for angle θ and then solve for y_m , we find

$$y_m = \frac{m\lambda D}{d}. \quad (35.2.6)$$

For the next maximum as we move away from the pattern's center, we have

$$y_{m+1} = \frac{(m+1)\lambda D}{d}. \quad (35.2.7)$$

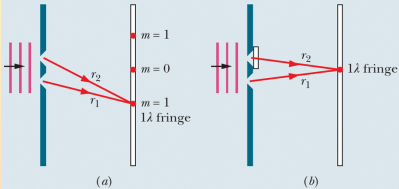
We find the distance between these adjacent maxima by subtracting Eq. 35.2.6 from Eq. 35.2.7:

$$\begin{aligned} \Delta y &= y_{m+1} - y_m = \frac{\lambda D}{d} \\ &= \frac{(546 \times 10^{-9} \text{ m})(55 \times 10^{-2} \text{ m})}{0.12 \times 10^{-3} \text{ m}} \\ &= 2.50 \times 10^{-3} \text{ m} \approx 2.5 \text{ mm}. \end{aligned} \quad (\text{Answer})$$

As long as d and θ in Fig. 35.2.5a are small, the separation of the interference fringes is independent of m ; that is, the fringes are evenly spaced.

Problem–double slit interference with a phase shift

The difference in indexes causes a phase shift between the rays, moving the 1λ fringe upward.



A double slit interference pattern is produced on a screen by a monochromatic light at a wavelength of 600 nm. A strip of transparent plastic with index of refraction $n = 1.50$ is to be placed over one of the slits. Its presence changes the interference between the light waves from the two slits, causing the interference pattern to be shifted up or down.

Figure (a) at left shows the original locations of the fringes. The purpose of the plastic is to shift the pattern upwards so that the lower $m = 1$ bright fringe is shifted to the centre of the pattern, as shown in (b).

Should the plastic be placed over the top slit (as arbitrarily shown in figure (b)) or the bottom slit, and what thickness L should it have?

Solution

Should the plastic be placed over the top slit (as arbitrarily drawn in example 35.3 figure 1b) or the bottom slit, and what thickness L should it have?

KEY IDEA

The interference at a point on the screen depends on the phase difference of the light rays arriving from the two slits. The light rays are in phase at the slits because they derive from the same wave, but their relative phase can shift on the way to the screen due to (1) a difference in the length of the paths they follow and (2) a difference in the number of their internal wavelengths λ_n in the materials through which they pass. The first condition applies to any off-centre point, and the second condition applies when the plastic covers one of the slits.

Path length difference: Example 35.3 figure 1a shows rays r_1 and r_2 along which waves from the two slits travel to reach the lower $m = 1$ bright fringe. Those waves start in phase at the slits but arrive at the fringe with a phase difference of exactly 1 wavelength. To remind ourselves of this main characteristic of the fringe, let us call it the 1λ fringe. The one-wavelength phase difference is due to the one-wavelength path length difference between the rays reaching the fringe; that is, there is exactly one more wavelength along ray r_2 than along r_1 .

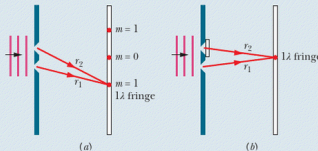
Example 35.3 figure 1b shows the 1λ fringe shifted up to the centre of the pattern with the plastic strip over the top slit (we still do not know whether the plastic should be there or over the bottom slit). The figure also shows the new orientations of rays r_1 and r_2 to reach that fringe. There still must be one more wavelength along r_2 than along r_1 (because they still produce the 1λ fringe), but now the path length difference between those rays is zero, as we can tell from the geometry of example 35.3 figure 1a. However, r_2 now passes through the plastic.

Internal wavelength: The wavelength λ_n of light in a material with index of refraction n is smaller than the wavelength in vacuum, as given by equation 35.11 ($\lambda_n = \lambda/n$). Here, this means that the wavelength of the light is smaller in the plastic than in the air. Thus, the ray that passes through the plastic will have more wavelengths along it than the ray that passes through only air — so we do get the one extra wavelength we need along ray r_2 by placing the plastic over the top slit, as drawn in example 35.3 figure 1b.

EXAMPLE 35.3 FIGURE 1

(a) Arrangement for two-slit interference (not to scale). The locations of three bright fringes (or maxima) are indicated. (b) A strip of plastic covers the top slit. We want the 1λ fringe to be at the centre of the pattern.

The difference in indexes causes a phase shift between the rays, moving the 1λ fringe upward.



Thickness: To determine the required thickness L of the plastic, we first note that the waves are initially in phase and travel equal distances L through different materials (plastic and air). Because we know the phase difference and require L , we use equation 35.16,

$$N_2 - N_1 = \frac{L}{\lambda} (n_2 - n_1). \quad (35.3.1)$$

We know that $N_2 - N_1$ is 1 for a phase difference of one wavelength, n_2 is 1.50 for the plastic in front of the top slit, n_1 is 1.00 for the air in front of the bottom slit, and λ is 600×10^{-9} m. Then equation 35.3.1 tells us that, to shift the lower $m = 1$ bright fringe up to the centre of the interference pattern, the plastic must have the thickness

$$L = \frac{\lambda (N_2 - N_1)}{n_2 - n_1} = \frac{(600 \times 10^{-9} \text{ m})}{1.50 - 1.00} = 1.2 \times 10^{-6} \text{ m}.$$

(Answer)

Problem—combine three light waves using phasors

Three light waves
combine at a certain
point where their electric
field magnitudes are

$$E_1 = E_0 \sin \omega t,$$

$$E_2 = E_0 \sin(\omega t + 60^\circ),$$

$$E_3 = E_0 \sin(\omega t - 30^\circ),$$

Find the resultant
electric field $E(t)$ at that
point.

Solution

Sample Problem 35.3.1 Combining three light waves by using phasors

Three light waves combine at a certain point where their electric field components are

$$E_1 = E_0 \sin \omega t,$$

$$E_2 = E_0 \sin(\omega t + 60^\circ),$$

$$E_3 = E_0 \sin(\omega t - 30^\circ).$$

Find their resultant component $E(t)$ at that point.

KEY IDEA

The resultant wave is

$$E(t) = E_1(t) + E_2(t) + E_3(t).$$

We can use the method of phasors to find this sum, and we are free to evaluate the phasors at any time t .

Calculations: To simplify the solution, we choose $t = 0$, for which the phasors representing the three waves are shown in Fig. 35.3.3. We can add these three phasors either directly on a vector-capable calculator or by components. For the component approach, we first write the sum of their horizontal components as

$$\sum E_h = E_0 \cos 0 + E_0 \cos 60^\circ + E_0 \cos(-30^\circ) = 2.37E_0.$$

The sum of their vertical components, which is the value of E at $t = 0$, is

$$\sum E_v = E_0 \sin 0 + E_0 \sin 60^\circ + E_0 \sin(-30^\circ) = 0.366E_0.$$

The resultant wave $E(t)$ thus has an amplitude E_R of

$$E_R = \sqrt{(2.37E_0)^2 + (0.366E_0)^2} = 2.4E_0,$$

and a phase angle β relative to the phasor representing E_1 of

$$\beta = \tan^{-1} \left(\frac{0.366E_0}{2.37E_0} \right) = 8.8^\circ.$$

We can now write, for the resultant wave $E(t)$,

$$\begin{aligned} E &= E_R \sin(\omega t + \beta) \\ &= 2.4E_0 \sin(\omega t + 8.8^\circ). \quad (\text{Answer}) \end{aligned}$$

Be careful to interpret the angle β correctly in Fig. 35.3.3: It is the constant angle between E_R and the phasor representing E_1 as the four phasors rotate as a single unit around the origin. The angle between E_R and the horizontal axis in Fig. 35.3.3 does not remain equal to β .

Phasors that represent waves can be added to find the net wave.

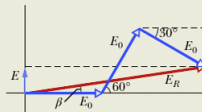


Figure 35.3.3 Three phasors, representing waves with equal amplitudes E_0 and with phase constants 0° , 60° , and -30° , shown at time $t = 0$. The phasors combine to give a resultant phasor with magnitude E_R , at angle β .

Did Young's double slit experiment *really* rule out particles?



<https://www.youtube.com/watch?v=GzbKb59my3U>

We now know that light comes in indivisible units called photons.

If you direct a beam of photons towards a double slit such that only one photon is in transit from the light source to the screen at any one time, what would you see?

How about a beam of electrons?

See <https://www.youtube.com/watch?v=ZJ-0PBRuthc>

The takeaway point here is that photons, and particles such as electrons (or in fact even macroscopic collections particles, such as balls and humans), have *both* wave-like properties *and* particle-like properties. They are “things” that obey the laws of quantum mechanics. “Waves” and “particles” are just mental images that help up understand the physics—however, they can also mislead.