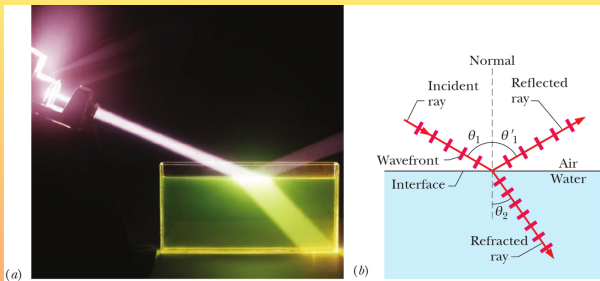


Lecture 10—learning outcomes

- Recognise that light can be considered as both a wave and a particle.
- Use Huygens principle to describe the propagation of light.
- Recognise when the ray approximation of light is valid.
- Use the law of reflection to predict the path of a light ray.
- Use the law of refraction to predict the path of a light ray.
- Calculate the relative refractive index of a medium given the angle of incidence and of refraction or the speed of the wave in two media.
- Explain how rainbows are created.
- Explain what is meant for total internal reflection and calculate the critical angle given suitable data

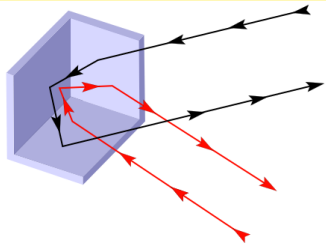
Textbook chapters: 33.5–6, 35.1 <https://ebookcentral.proquest.com/lib/unsw/reader.action?c=UERG&docID=30765671&ppg=802>

Laws of reflection and refraction

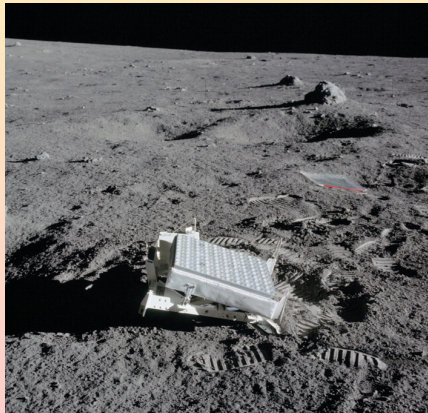
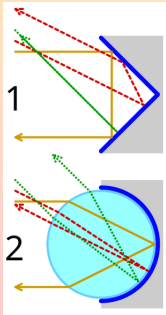


- **The Law of Reflection:** A reflected ray lies in the plane of incidence and has an angle of reflection equal to the angle of incidence, both relative to the normal. $\theta'_1 = \theta_1$
- **The Law of Refraction:** A refracted ray lies in the plane of incidence and has an angle of refraction θ_2 that is related to the angle of incidence θ_1 by $n_2 \sin \theta_2 = n_1 \sin \theta_1$ (Snell's Law)

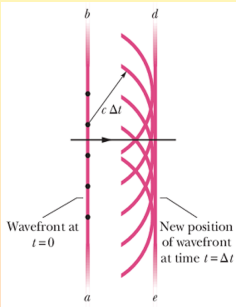
What is a retroreflector?



- The figure at the left show a corner retroreflector. If light hits the retroreflector in such a way as to bounce off at least two surfaces, it will be reflected back to its source.
- The figure at bottom left shows a 2D corner retroreflector and a cat's eye retroreflector.
- The figure below is a retroreflector on the Moon.



Huygens' Principle



- Huygens' Principle: *"All points on a wavefront serve as point sources of spherical secondary wavelets. After a time t , the new position of the wavefront will be that of a surface tangent to the secondary wavelets."* Dates from 1678.
- The law of refraction can be derived from Huygens' Principle by assuming that the index of refraction n of any medium is

$$n = \frac{c}{v}$$

in which v is the speed of light in the medium and c is the speed of the light in a vacuum. The wavelength of light λ_n in the medium depends on the index of a fraction n of the medium:

$$\lambda_n = \frac{\lambda}{n}$$

- Because of this dependency, the phase difference between two waves can change if they pass through different materials with different indices of refraction.

The index of refraction, n

Table 33-1 Some Indexes of Refraction^a

Medium	Index	Medium	Index
Vacuum	Exactly 1	Typical crown glass	1.52
Air (STP) ^b	1.00029	Sodium chloride	1.54
Water (20°C)	1.33	Polystyrene	1.55
Acetone	1.36	Carbon disulfide	1.63
Ethyl alcohol	1.36	Heavy flint glass	1.65
Sugar solution (30%)	1.38	Sapphire	1.77
Fused quartz	1.46	Heaviest flint glass	1.89
Sugar solution (80%)	1.49	Diamond	2.42

^aFor a wavelength of 589 nm (yellow sodium light).

^bSTP means “standard temperature (0°C) and pressure (1 atm).”

Light as a wave

Refraction occurs at the surface, giving a new direction of travel.

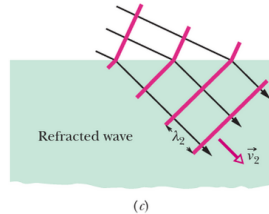
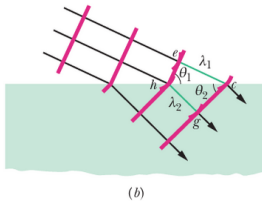
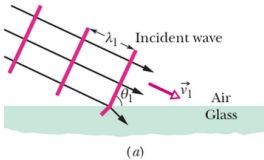
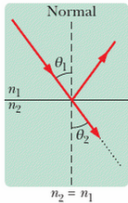


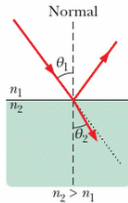
Figure 35-3 The refraction of a plane wave at an air–glass interface, as portrayed by Huygens’ principle. The wavelength in glass is smaller than that in air. For simplicity, the reflected wave is not shown. Parts (a) through (c) represent three successive stages of the refraction.

Refraction of light across an interface between materials



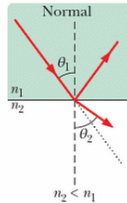
(a)

If the indexes match, there is no direction change.



(b)

If the next index is greater, the ray is bent *toward* the normal.

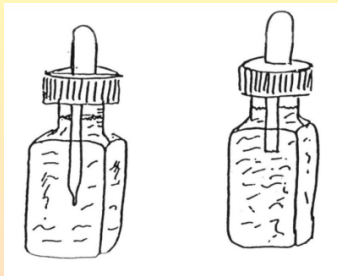


(c)

If the next index is less, the ray is bent *away from* the normal.

- (a) The beam does not bend when the refractive indexes n_1 and n_2 are equal.
- (b) The beam moves *towards* the normal when $n_2 > n_1$.
- (c) The beam moves *away from* the normal when $n_2 < n_1$.

Demo: La5 - Disappearing Dropper

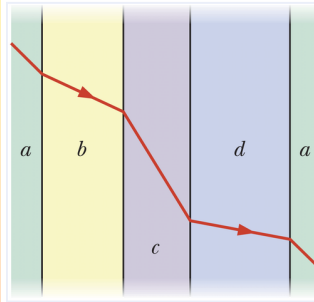


- A dropping bottle is filled with methyl benzoate.
- As methyl benzoate has nearly the same refractive index as that of the glass in the dropper, when the methyl benzoate is drawn up into the dropper the dropper seems to disappear.
- Even if you know how this works, it is still impressive.

Mini-quiz on refractive index



Light rays passing through slabs with different refractive indices



Rank the slabs according to their refractive index, in ascending order (lowest refractive index at the top) by drag and dropping the letters:

- *b*
- *d*
- *a*
- *c*

Submit for marking

What actually is light?

- Light appears to be well described as **electromagnetic waves** that satisfy Maxwell's equations. In the visible part of the electromagnetic spectrum, the colour of the light is determined by the frequency of the waves.
- We can represent waves with **wavefronts** using Huygens' Principle.
- We can also represent waves as **rays** that are in the direction of propagation, perpendicular to the wavefronts.
- The path followed by a ray between two points is given by Fermat's Principle: *the path taken by a ray between two given points is the path that can be traveled in the least time*. Dates from 1662.
- Many experiments show that light appears to come in discrete lumps and behaves like **particles**, called a photons.
- So, is light a wave or a particle? The best answer is probably “neither”! Light is just light, and comparing it to our familiar concepts of waves or particles is just a helpful analogy.

Refraction analogy



Refraction analogy, continued...

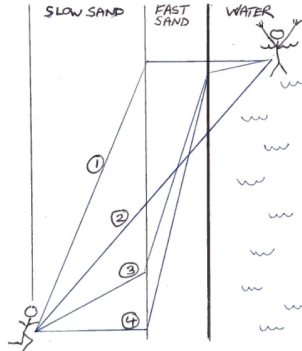


- Suppose you see a person in trouble in the water, 5m from the beach.
- You are some distance away, where the sand makes running slow.
- There is a 5m width of wet sand close to the water where the running is easy.
- What is the optimum path to follow to reach the person as quickly as possible?

Mini-quiz on a refraction analogy



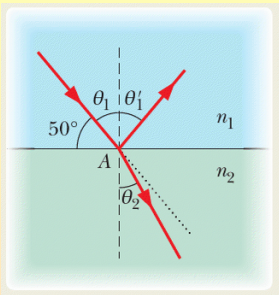
Refraction analogy



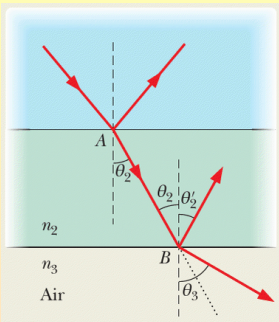
Which of the four paths is the fastest to reach the drowning person?

- ☐ 1
- ☐ 2
- ☐ 3
- ☐ 4

Submit for marking



- A beam of monochromatic light reflects and refracts at point A on the interface between material 1 with a refractive index of 1.33 and material 2 with index of refraction 1.77. The incident beam makes an angle of 50° with the interface. What is the angle of reflection at point A ? What is the angle of refraction there?



- The light that enters material 2 at point A then reaches point B on the interface between material 2 and material 3, which is air, as shown in the figure. The interface through B is parallel to that through A . At B , some of the light reflects and the rest enters the air. What is the angle of reflection? What is the angle of refraction into the air?

(a) In Fig. 33-22a, a beam of monochromatic light reflects and refracts at point A on the interface between material 1 with index of refraction $n_1 = 1.33$ and material 2 with index of refraction $n_2 = 1.77$. The incident beam makes an angle of 50° with the interface. What is the angle of reflection at point A ? What is the angle of refraction there?

KEY IDEAS

(1) The angle of reflection is equal to the angle of incidence, and both angles are measured relative to the normal to the surface at the point of reflection. (2) When light reaches the interface between two materials with different indexes of refraction (call them n_1 and n_2), part of the light can be refracted by the interface according to Snell's law, Eq. 33-40:

$$n_2 \sin \theta_2 = n_1 \sin \theta_1, \quad (33-42)$$

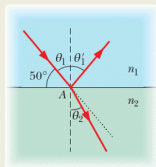
where both angles are measured relative to the normal at the point of refraction.

Calculations: In Fig. 33-22a, the normal at point A is drawn as a dashed line through the point. Note that the angle of incidence θ_1 is not the given 50° but is $90^\circ - 50^\circ = 40^\circ$. Thus, the angle of reflection is

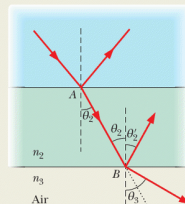
$$\theta_1' = \theta_1 = 40^\circ. \quad (\text{Answer})$$

The light that passes from material 1 into material 2 undergoes refraction at point A on the interface between the two materials. Again we measure angles between light rays and a normal, here at the point of refraction. Thus, in Fig. 33-22a, the angle of refraction is the angle marked θ_2 . Solving Eq. 33-42 for θ_2 gives us

$$\begin{aligned} \theta_2 &= \sin^{-1} \left(\frac{n_1}{n_2} \sin \theta_1 \right) = \sin^{-1} \left(\frac{1.33}{1.77} \sin 40^\circ \right) \\ &= 28.88^\circ \approx 29^\circ. \end{aligned} \quad (\text{Answer})$$



(a)



(b)

Figure 33-22 (a) Light reflects and refracts at point A on the interface between materials 1 and 2. (b) The light that passes through material 2 reflects and refracts at point B on the interface between materials 2 and 3 (air). Each dashed line is a normal. Each dotted line gives the incident direction of travel.

This result means that the beam swings toward the normal (it was at 40° to the normal and is now at 29°). The reason is that when the light travels across the interface, it moves into a material with a greater index of refraction. **Caution:** Note that the beam does *not* swing through the normal so that it appears on the left side of Fig. 33-22a.

(b) The light that enters material 2 at point A then reaches point B on the interface between material 2 and material 3, which is air, as shown in Fig. 33-22b. The interface through B is parallel to that through A . At B , some of the light reflects and the rest enters the air. What is the angle of reflection? What is the angle of refraction into the air?

Calculations: We first need to relate one of the angles at

point B with a known angle at point A . Because the interface through point B is parallel to that through point A , the incident angle at B must be equal to the angle of refraction θ_2 , as shown in Fig. 33-22*b*. Then for reflection, we again use the law of reflection. Thus, the angle of reflection at B is

$$\theta'_2 = \theta_2 = 28.88^\circ \approx 29^\circ. \quad (\text{Answer})$$

Next, the light that passes from material 2 into the air undergoes refraction at point B , with refraction angle θ_3 . Thus, we again apply Snell's law of refraction, but this time we write Eq. 33-40 as

$$n_3 \sin \theta_3 = n_2 \sin \theta_2. \quad (33-43)$$

Solving for θ_3 then leads to

$$\begin{aligned} \theta_3 &= \sin^{-1} \left(\frac{n_2}{n_3} \sin \theta_2 \right) = \sin^{-1} \left(\frac{1.77}{1.00} \sin 28.88^\circ \right) \\ &= 58.75^\circ \approx 59^\circ. \quad (\text{Answer}) \end{aligned}$$

Thus, the beam swings away from the normal (it was at 29° to the normal and is now at 59°) because it moves into a material (air) with a lower index of refraction.

Optical path length (OPL)

The difference in indexes causes a phase shift between the rays.

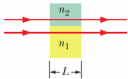


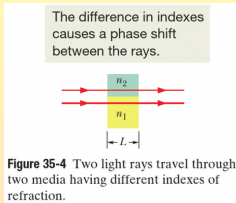
Figure 35-4 Two light rays travel through two media having different indexes of refraction.

- The figure shows two rays travelling from left to right. Assume the waves have identical wavelengths λ that are initially in phase in a vacuum. One of the waves travel through a medium with refractive index n_1 and length L . The other travels through a medium with index refraction n_2 and the same length. When the waves leave the two media, they will have the same wavelength they originally had in the vacuum. However, because their wavelengths differed in the two media, the two waves may no longer be in phase.
- The **optical path length** is *the distance light would have to travel through a vacuum to achieve the same phase as the light if it traveled distance d through a medium of refractive index n .*

$$\text{OPL} = nd$$

- In the figure, the phase difference (in units of wavelengths in a vacuum) between the two rays is $\Delta\text{OPL}/\lambda = (n_2 - n_1)L/\lambda$.
- If the phase difference is an integer, then the rays interfere constructively (maximum brightness). If the phase difference is an integer $+0.5$, then the rays interfere destructively (darkness).

Example question: phase difference



- The two light waves in the figure, represented by rays, have a wavelength of 550.0 nm before entering the media. They also have equal amplitudes and are in phase. Medium 1 is air, and medium 2 is a transparent plastic layer of index of refraction 1.600 and thickness 2.600 μm . What is the phase difference of the emerging waves in wavelengths, radians, and degrees? What is their effective phase difference in wavelengths?
- If the waves reach the same point on a distant screen, what type of interference would they produce?

Sample Problem 35.01 Phase difference of two waves due to difference in refractive indexes

In Fig. 35-4, the two light waves that are represented by the rays have wavelength 550.0 nm before entering media 1 and 2. They also have equal amplitudes and are in phase. Medium 1 is now just air, and medium 2 is a transparent plastic layer of index of refraction 1.600 and thickness 2.600 μm .

(a) What is the phase difference of the emerging waves in wavelengths, radians, and degrees? What is their effective phase difference (in wavelengths)?

KEY IDEA

The phase difference of two light waves can change if they travel through different media, with different indexes of refraction. The reason is that their wavelengths are different in the different media. We can calculate the change in phase difference by counting the number of wavelengths that fits into each medium and then subtracting those numbers.

Calculations: When the path lengths of the waves in the two media are identical, Eq. 35-9 gives the result of the subtraction. Here we have $n_1 = 1.000$ (for the air), $n_2 = 1.600$, $L = 2.600 \mu\text{m}$, and $\lambda = 550.0 \text{ nm}$. Thus, Eq. 35-9 yields

$$\begin{aligned} N_2 - N_1 &= \frac{L}{\lambda} (n_2 - n_1) \\ &= \frac{2.600 \times 10^{-6} \text{ m}}{5.500 \times 10^{-7} \text{ m}} (1.600 - 1.000) \\ &= 2.84. \end{aligned} \quad (\text{Answer})$$

Thus, the phase difference of the emerging waves is 2.84 wavelengths. Because 1.0 wavelength is equivalent to 2π rad and 360° , you can show that this phase difference is equivalent to

$$\text{phase difference} = 17.8 \text{ rad} \approx 1020^\circ. \quad (\text{Answer})$$

The effective phase difference is the decimal part of the actual phase difference *expressed in wavelengths*. Thus, we have

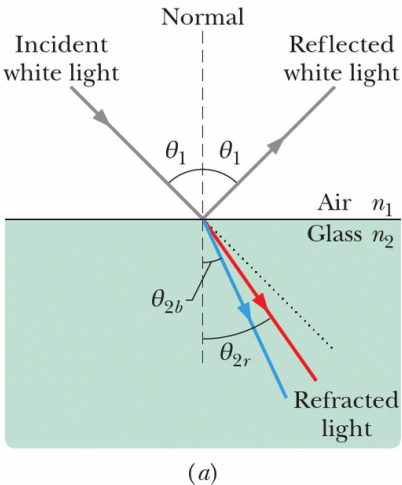
$$\text{effective phase difference} = 0.84 \text{ wavelength}. \quad (\text{Answer})$$

You can show that this is equivalent to 5.3 rad and about 300° . **Caution:** We do *not* find the effective phase difference by taking the decimal part of the actual phase difference as expressed in radians or degrees. For example, we do *not* take 0.8 rad from the actual phase difference of 17.8 rad.

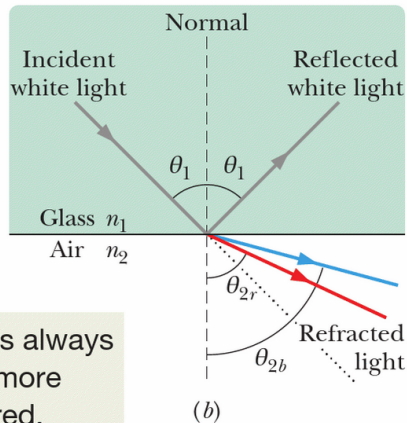
(b) If the waves reached the same point on a distant screen, what type of interference would they produce?

Reasoning: We need to compare the effective phase difference of the waves with the phase differences that give the extreme types of interference. Here the effective phase difference of 0.84 wavelength is between 0.5 wavelength (for fully destructive interference, or the darkest possible result) and 1.0 wavelength (for fully constructive interference, or the brightest possible result), but closer to 1.0 wavelength. Thus, the waves would produce intermediate interference that is closer to fully constructive interference — they would produce a relatively bright spot.

Refraction varies with wavelength



Blue is always bent more than red.



Chromatic dispersion

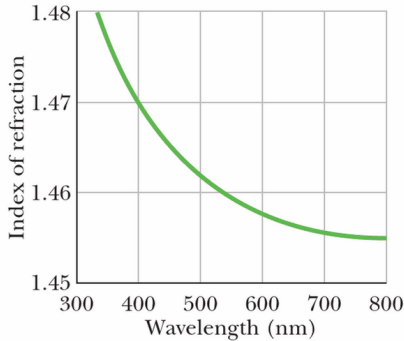


Figure 33-18 The index of refraction as a function of wavelength for fused quartz. The graph indicates that a beam of short-wavelength light, for which the index of refraction is higher, is bent more upon entering or leaving quartz than a beam of long-wavelength light.

Total internal reflection

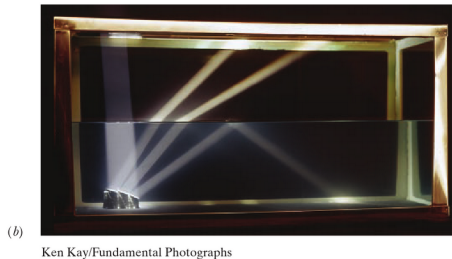
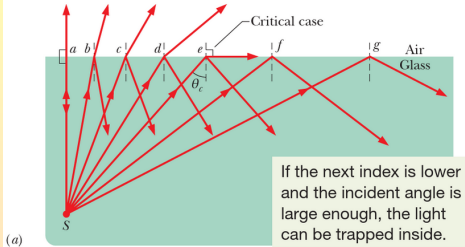


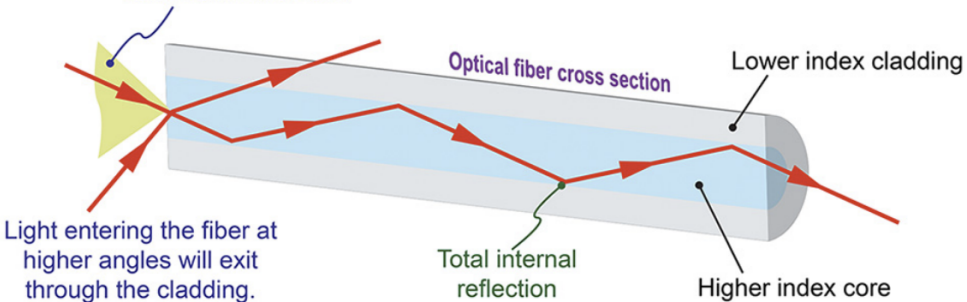
Figure 33-23 (a) Total internal reflection of light from a point source S in glass

- From Snell's Law, $n_1 \sin \theta_1 = n_2 \sin \theta_2$, there will be a critical angle, θ_c , at which the angle of the ray leaving the high refractive medium will be 90° .
- At angles greater than or equal to the critical angle θ_c , the light undergoes total internal reflection

$$\theta_c = \sin^{-1} \frac{n_2}{n_1}$$

Basic Operation of an Optical Fiber

Light entering the fiber within this angular range will undergo total internal reflection and travel down the fiber.



Formation of rainbows

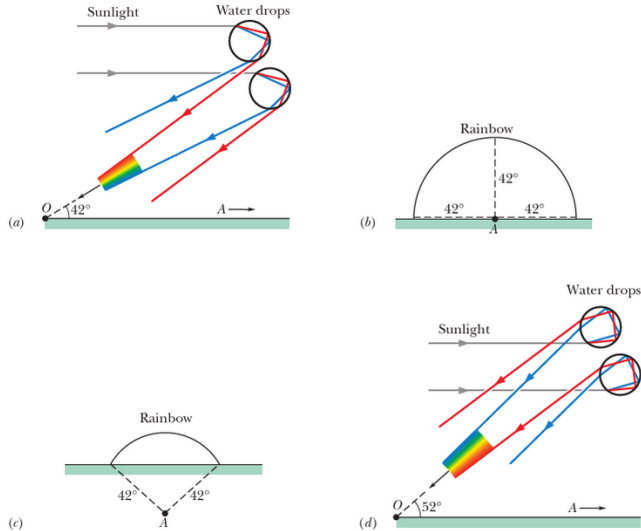


Figure 33-21 (a) The separation of colors when sunlight refracts into and out of falling raindrops leads to a primary rainbow. The antisolar point A is on the horizon at the right. The rainbow colors appear at an angle of 42° from the direction of A . (b) Drops at 42° from A in any direction can contribute to the rainbow. (c) The rainbow arc when the Sun is higher (and thus A is lower). (d) The separation of colors leading to a secondary rainbow.

Rainbows



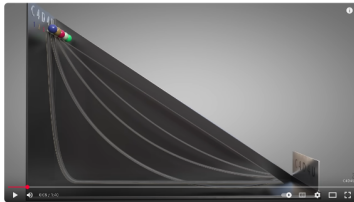
The URL is on the right edge of the image.

We will watch a few minutes of this video in the lecture, I *strongly* encourage you to watch the whole thing.

Mini-quiz on the brachistochrone



The fastest path between two points



The figure above shows 5 balls at the top of separate tracks leading to the bottom right. Assume that the balls will slide down the tracks without rolling when released. The balls are labelled 1 (blue), 2 (brown), 3 (red), 4 (grey), 5 (green).

When the balls have dropped by half the height of the ramp, which one will be travelling fastest?

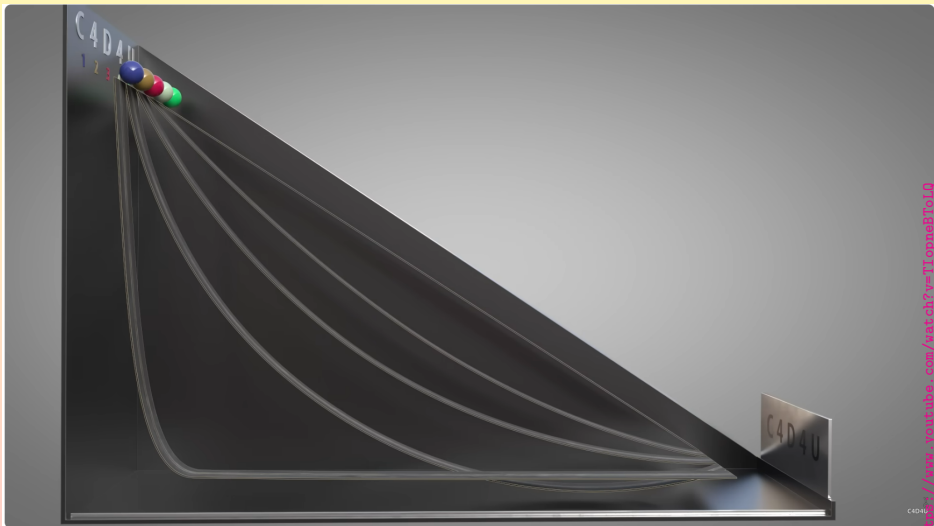
- ☐ 1 (blue) the steepest ramp
- ☐ 2 (brown) the next steepest
- ☐ 3 (red) the middle steepest
- ☐ 4 (grey) almost straight
- ☐ 5 (green) the straight ramp
- ☐ They will have the same speed

When the balls shoot off the end of the ramp, which one will be travelling fastest?

- ☐ 1 (blue)
- ☐ 2 (brown)
- ☐ 3 (red)
- ☐ 4 (grey)
- ☐ 5 (green)
- ☐ They will have the same speed

Which one will reach the end of the ramp first?

The brachistochrone

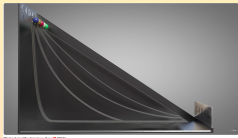


Which is faster * Brachistochrone Curve ♥ C4D4U

<https://www.youtube.com/watch?v=TIopne8ToLQ>

The URL is on the right edge of the image.

The brachistochrone—solved using Snell's Law!

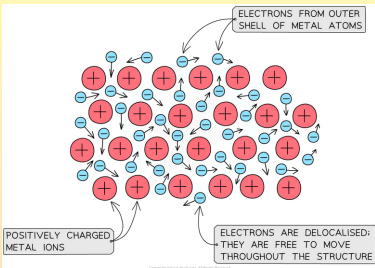


- The brachistochrone problem is to find the fastest possible path from the start to the end point.
- Light naturally follows the fastest path between two points. Varying the refractive index along the path varies the speed of the light and provides a way of simulating the situation of a ball falling under gravity.
- Note that all balls at the same height have the same speed since the change in gravitational PE is identical and appears as KE.
- So we need a refractive index distribution that depends (in an appropriate fashion) only on height, and we will have accurately simulated the ball problem.
- The brachistochrone is then found by applying Snell's Law along every point of the path.
- The solution turns out to be a cycloid function, which is also the curve traced by a point on the edge of a wheel.

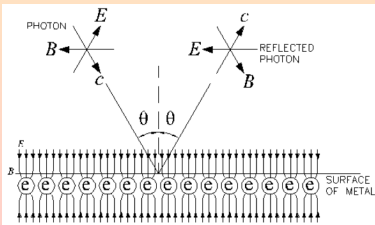
Fundamental questions remain...

- Why do metals reflect light, whereas non-metals usually absorb it?
- Why is glass transparent?
- Why does light slow down in glass?

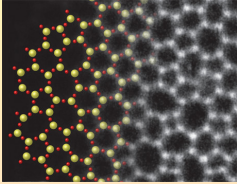
Why do metals reflect light?



- The electrons in a metal (shown in blue at the left) are free to move away from their parent atoms (shown in red) and roam throughout the metal.
- The ability of the electrons to move freely gives the metal high conductivity (both electrical and thermal).
- When an EM wave reaches the metal surface (bottom illustration at left), the electrons at the surface oscillate due to the forces acting on them.
- Each oscillating electron in turn emits EM radiation.
- Only in the reflection direction do all the EM waves add up to produce a reflected wave.
- The surface has to be flat at the scale of a fraction of λ for *specular* reflection to occur. Rougher surfaces can produce *diffuse* reflection.

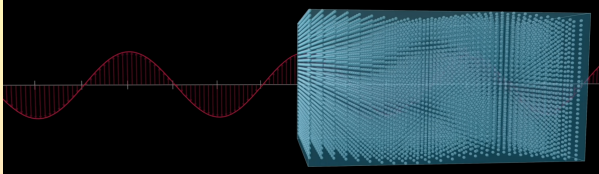


Why is glass transparent?



- Glass is an insulator, composed mostly of silicon dioxide molecules. See the electron microscope image on the right-hand half of the image. On the left-hand half, silicon atoms are shown in yellow, oxygen in red.
- The outer electrons (the ones furthest from the nuclei of the atoms of which glass is composed) are NOT free to roam—they have to stay close to their parent nucleus.
- Now, the energy that an electron in an atom has is restricted (quantised) to certain values.
- An electron can *only* absorb a photon if its energy corresponds to an allowed energy difference. Otherwise the photon doesn't interact, and just keeps on going.
- In glass, the energy levels are such that a photon at visible wavelengths *does not have enough energy* to excite an electron to the nearest higher energy level. High energy UV photons do have enough energy.
- So, glass is transparent to visible light, but opaque in the UV.

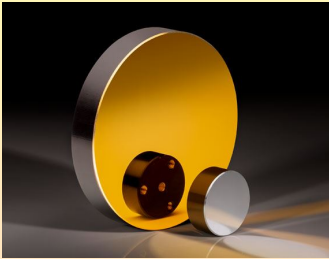
Why does light slow down in glass?



<https://youtu.be/KTzGBJPuJwM>

- As we have seen, the electrons in glass are not free to absorb photons, but they do oscillate slightly due to the force from the electromagnetic wave as it passes through.
- Every electron that oscillates also generates an EM wave that travels away from it *at the speed of light*.
- Every electron feels the combined fields from all the other electrons, suitably delayed by the light travel time from each one.
- Remarkably, the combined effect of all of these interacting oscillating electrons, and the incoming light, is to produce an EM wave just like the incoming one, but with a phase velocity of c/n .
- Since the frequency f of the light can't change, its wavelength $\lambda' = \lambda/n$.

Why is gold yellow?



- The figure at left shows two glass disks, one coated with a layer of gold, and the other with aluminium.
- The reflectivities of gold (Au), aluminium (Al), and silver (Ag) as a function of wavelength are shown in the lower figure.

- Gold appears yellow since it absorbs blue light.
- But why does it absorb blue light?

- The nucleus of a gold atom has more protons than aluminium or silver, so the inner (1s) electrons experience a greater attractive force and so have to orbit more quickly. In fact, they reach 58% of the speed of light! This causes an increase in the mass of the electrons via Einstein's Special Relativity, which leads to the energy levels being closer together—close enough that blue light can now be absorbed.

