Today’s lecture:

The Magnetic Field of a Solenoid

Magnetic Flux and Gauss’s Law

Faraday’s Law of Induction
The Magnetic Field of a Solenoid

A **solenoid** is a long wire wound around in the form of a helix.

A reasonably uniform magnetic field can be produced in the space surrounded by the turns of the wire – in other words, in the interior of the solenoid.

In that region, the field lines are:

- approximately parallel
- uniformly distributed
- close together

* i.e. the field is strong and almost uniform.
The magnetic field of a tightly wound solenoid (fig. a) is similar to that for a bar magnet (fig. b).

As the length of the solenoid increases:

• the interior field becomes more uniform
• the exterior field becomes weaker
An Ideal Solenoid - Characteristics

As the length of the solenoid becomes much greater than the radius of its turns, a solenoid with closely spaced turns approaches what we call an “ideal solenoid”

The field inside the ideal solenoid is uniform and strong.

The field outside the ideal solenoid is weak, but non-zero.

The field lines outside the ideal solenoid are circular, like those for a line of current.
Field in the interior of a Solenoid with Ampere’s Law

Apply Ampere’s law using loop 2 in the diagram to calculate the field inside the ideal solenoid.

Consider a rectangle with side $l$ parallel to the interior field and side $w$ perpendicular to the field.

The side of length $l$ inside the solenoid contributes to the field (path 1 in the diagram), while paths 2, 3 and 4 do not contribute.

\[ \oint B \cdot ds = \int B \cdot ds = B \int ds = Bl \]
Field in the interior of a Solenoid with Ampere’s Law

Total current through the rectangular path equals the current through each turn, multiplied by the number of turns.

\[ \therefore \int \mathbf{B} \cdot d\mathbf{s} = Bl = \mu_0 NI \]

Ampere’s law therefore gives:

\[ B = \mu_0 \frac{N}{l} I = \mu_0 nI \]

Where \( n = \frac{N}{l} \) is the number of turns per unit length.

Note – this is valid only at points near the centre of our long solenoid.
Consider a solenoid that is very long compared to its radius.

Of the following choices, the most effective way to increase the magnetic field in the interior of the solenoid is:

a) Double its length, keeping the number of turns per unit length constant.

b) Reduce its radius by half, keeping the turns per unit length constant.

c) Overwrapping the entire solenoid with an additional layer of current-carrying wire
Quick Answer

Consider a solenoid that is very long compared to its radius.

Of the following choices, the most effective way to increase the magnetic field in the interior of the solenoid is:

c) Overwrapping the entire solenoid with an additional layer of current-carrying wire

The magnetic field in a very long solenoid is independent of its length or radius.

Overwrapping with an additional layer of wire increases the number of turns per unit length, and so increases the magnetic field in the solenoid’s interior.
Ampere’s Law vs. Gauss’s Law

<table>
<thead>
<tr>
<th>Ampere’s Law</th>
<th>Gauss’s Law</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\oint B \cdot ds = \mu_0 I$</td>
<td>$\oint E \cdot dA = \frac{q}{\varepsilon_0}$</td>
</tr>
</tbody>
</table>

Integrals around a closed path vs. a closed surface – i.e. 2D vs. 3D geometrical figures.

The integrals are related to a fundamental constant multiplied by the source of the field.

For Gauss’s Law, we introduced the concept of *Electric Flux* (Lecture 3). Now we’re going to introduce *Magnetic Flux*. 
Magnetic Flux

The magnetic flux associated with a magnetic field is defined in a way similar to the way we defined the electric flux associated with an electric field.

Consider an area element $dA$ on an arbitrarily shaped surface.

The magnetic field in this element is $B$.

$dA$ is a vector that is perpendicular to the surface and has a magnitude equal to the area $dA$. 
Magnetic Flux

The magnetic flux, $\Phi_B$, is therefore:

$$\Phi_B = \int B \cdot dA$$

The unit of magnetic flux is $T.m^2 = Wb$

1 Wb is 1 weber
A special case is when a plane of area $A$ is placed in a magnetic field, $B$, which makes an angle $\theta$ with $dA$.

In this case, the magnetic flux is: $\Phi_B = BA \cos \theta$

In the case shown to the right, the field is parallel to the plane, (i.e. $\theta = 90^\circ$), which results in $\Phi_B = 0$.

The flux is maximum when the field is perpendicular to the plane (i.e. $\theta = 0^\circ$)
Gauss’s Law in Magnetism

Remember: Magnetic fields do not begin or end at any point.

In other words, they form closed loops, with the number of lines entering a surface equalling the number of lines leaving that surface.

Gauss’s Law in magnetism therefore says that the magnetic flux through any closed surface is always zero.

\[ \Phi_B = \oint \mathbf{B} \cdot d\mathbf{A} = 0 \]

Note: This would not be true if magnetic monopoles were found. It may be that they do not exist – although some theories predict that they do!
Gauss’s Law in Magnetism

Remember: Magnetic fields do not begin or end at any point.

In other words, they form closed loops, with the number of lines entering a surface equalling the number of lines leaving that surface.

Gauss’s Law in magnetism therefore says that the magnetic flux through any closed surface is always zero.

i.e. \[ \Phi_B = \oint B \cdot dA = 0 \]

Note: This would not be true if magnetic monopoles were found. It may be that they do not exist – although some theories predict that they do!
Gauss’s Law in Magnetism: One of Maxwell’s Equations.

The equations on the scary mug are Maxwell’s Equations...

The first is just a different way of stating Gauss’s law for electric fields.

The second is a restatement of Gauss’s law for magnetic fields.

The third is Faraday’s law of induction, which we’ll cover shortly...

The fourth is related to Ampere’s law...

Don’t worry about these equations for now, though – they’re fun for another course! 😊
Quick Quiz

Tired of studying physics, but wanting to use what she has learned in her future career, a promising student leaves UNSW to become a **super-villain**.

She decides that she wants to cancel the Earth’s magnetic field by running an enormous current loop around the Earth’s magnetic equator.

Which way should our villainess direct her current loop?

a) East to West

b) West to East
Quick Answer

Which way should our villainess direct her current loop?

b) **West to East**

The lines of Earth’s magnetic field enter the planet near the north geographic pole (around Hudson Bay, in Canada), and emerge from Antarctica. The field lines resulting from the current would therefore have to go in the opposite direction, to cancel that magnetic field out.
Michael Faraday

1791 – 1867

British physicist and chemist, with little formal education...

Great experimental scientist

Contributions to early electricity include:

The invention of the motor, generator and transformer

Electromagnetic induction (today’s lecture!)

The laws of electrolysis
The Faraday Cage – a brief aside…

Faraday demonstrated that the charge resided only on the exterior of a charged conductor, and that there is no field inside a conductor.

*Does this sound familiar?*

We describe such a setup as a “Faraday Cage”

A car acts as a Faraday Cage in a thunderstorm.

*Here’s a video of Richard Hammond using this fact to shoot lightning from his fingertips, courtesy of the BBC...*
The electromotive force (emf) is the voltage generated either by a battery or by the magnetic force (which we’ll come to in a moment).

For a battery, the emf is the maximum possible voltage that the battery can provide between its terminals.

The emf supplies energy – it does not apply a force (despite the name!).

Technically, the emf is classified as the external work expended per unit of charge to produce an electric potential difference across two terminals.
Induced Fields

Magnetic fields can vary as a function of time.

*e.g. the field around a wire before and after you turn the current on.*

Experiments by Faraday and Joseph Henry, conducted in 1831, showed that an emf could be induced in a circuit by a changing magnetic field.

The results of these experiments led directly to *Faraday’s Law of Induction.*
EMF Produced by a Changing Magnetic Field

A loop of wire is connected to a sensitive ammeter.

The ammeter measures the current flowing through the loop.

When a magnet is moved towards the loop, the ammeter shows that a current begins to flow in the loop.

In the figure to the right, the direction of the current was arbitrarily chosen to be negative.
If the magnet is then held stationary, the current through the loop quickly falls to zero, and remains there whilst the magnet is held stationary.

There is therefore no induced current at that time, despite the fact that the magnet is in the loop.

The induced current is only there when the magnet is moving.
EMF Produced by a Changing Magnetic Field

When the magnet is moved away from the loop, a current is once again induced in the loop.

The direction of the current in this case is in the opposite direction to that when the magnet was inserted to the loop.

The ammeter in this case therefore registers a positive value.
Induced Current Experiment - Summary

When a magnet is moved toward a loop of wire connected to a sensitive ammeter, the ammeter shows that a current is induced in the loop.

When the magnet is held stationary, there is no induced current in the loop, even when the magnet is inside the loop.

When the magnet is moved away from the loop, the ammeter shows that the induced current is opposite that shown in part a.
The ammeter deflects when the magnet is moving towards or away from our loop.

The ammeter also deflects when the loop is moved towards or away from the magnet.

Therefore, the loop detects that the magnet is moving relative to it.

We relate this detection to a change in the magnetic field.

We measure the induced current that is produced by an induced emf.
Induction

An induced current is produced by a changing magnetic field.

There is an induced emf associated with the induced current.

This means that it is possible to produce a current without a battery being present in a circuit.

Faraday’s law of induction describes the induced emf.
Faraday’s Experiment

A primary wire is connected to a switch and a battery.

The wire is then wrapped around an iron ring.

This makes a primary coil.

A secondary coil is also wrapped around the iron ring, but this secondary coil is not connected to a battery. Nor is it connected to the primary.

An ammeter reads the current through wire 2.
Faraday’s Experiment

What happens when the switch is closed?

At the instant the switch is closed, the ammeter changes from zero in one direction, then returns to zero.

When the switch is reopened, the ammeter changes in the opposite direction, then returns to zero.

The ammeter reads zero when there is a steady current, or when there is no current in the primary circuit.
Faraday's Experiment - Conclusions

An electric current can be induced in a loop by a changing magnetic field.

In the experimental setup described on the last two slides, this would be the current measured in the secondary circuit.

The induced current exists only while the magnetic field through the loop is changing.

This is generally expressed as: **an induced emf is produced in the loop by the changing magnetic field.**

*Note: the existence of the magnetic flux is not sufficient to produce the induced emf. Instead, the flux through the loop must be changing.*
Faraday’s Law of Induction

The emf induced in a circuit is directly proportional to the time rate of change of the magnetic flux through the circuit.

Mathematically, we express this as:

\[ \varepsilon = - \frac{d\Phi_B}{dt} \]
Faraday’s Law of Induction

\[ \varepsilon = -\frac{d\Phi_B}{dt} \]

Remember that \( \Phi_B \) is the magnetic flux through the circuit, and is found by:

\[ \Phi_B = \int B \cdot dA \]

If our circuit consists of \( N \) loops, each of which has the same area, and if \( \Phi_B \) is the flux through a single loop, then an emf is induced in every loop, and Faraday’s law becomes:

\[ \varepsilon = -N \frac{d\Phi_B}{dt} \]
Faraday’s Law: An Example

Assume we have a loop that encloses an area $A$, and lies in a uniform magnetic field $B$.

From earlier, the magnetic flux through the loop is:

$$\Phi_B = BA \cos \theta$$

The induced emf is therefore:

$$\varepsilon = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}(BA \cos \theta)$$
Ways to induce an emf

For our previous example:

\[ \varepsilon = - \frac{d\Phi_B}{dt} = - \frac{d}{dt} (BA \cos \theta) \]

Clearly, an emf, \( \varepsilon \), would be induced if:

- The magnitude of B was to change with time.
- The area enclosed, A, was to change with time.
- The angle, \( \theta \), was to change with time.
- Or, if any combination of the above were to occur.
Quick Quiz

A circular loop of wire is held in a uniform magnetic field, with the plane of the loop perpendicular to the field lines.

Which of the following will not cause a current to be induced in the loop?

a) Crushing the loop

b) Rotating the loop about an axis perpendicular to the field lines

c) Rotating the loop about an axis parallel to the field lines

d) Keeping the orientation of the loop fixed, and moving it along the field lines.

e) Pulling the loop out of the field
Quick Answer

A circular loop of wire is held in a uniform magnetic field, with the plane of the loop perpendicular to the field lines.

Which of the following will not cause a current to be induced in the loop?

- c) Rotating the loop about an axis parallel to the field lines
- d) Keeping the orientation of the loop fixed, and moving it along the field lines.

In all of the other cases there is a change in the magnetic flux through the loop, and so a current would be induced in the loop.
Quick Quiz

The figure below shows the strength versus time for a magnetic field that passes through a fixed loop. The magnitude of the magnetic field at any time is uniform over the area of the loop.

Rank the magnitudes of the emf generated in the loop at the five instants indicated, from largest to smallest.

a) a, b, c, d  
b) b, d, a, c  
c) c, d, b, a  
d) d, c, a, b  
e) e, a, d, c
Quick Answer

Rank the magnitudes of the emf generated in the loop at the five instants, from largest to smallest.

c) c, d, b, a

Specifically c, d=e, b, a.

The magnitude of the emf is proportional to the rate of change of the magnetic flux. This is proportional to the rate of change of the magnetic field – i.e. the change in the slope of the graph.

The magnitude of the slope is largest at c. Points d and e are on a straight line, so the slope is the same at each. Point b represents a point of relatively small slope, while a is at a point of zero slope, since the line is horizontal at that point...
Quick Quiz

Suppose that you want to steal power for your home from the electricity company by placing a loop of wire near a transmission cable, so as to induce an emf in the cable (note: Doing this would be illegal!).

You would have to:

a) Place your loop so that the transmission cable passes through the loop

b) Simply place your loop near to the transmission cable
Quick Answer

Suppose that you want to steal power for your home from the electricity company by placing a loop of wire near a transmission cable, so as to induce an emf in the cable (note: Doing this would be illegal!).

You would have to:

b) Simply place your loop near to the transmission cable

The magnetic field lines around the transmission cable will be circular, centred on the cable. If you place your loop around the cable, there are no field lines passing through the loop, so no emf is induced.

The loop must be placed next to the cable, with the plane of the loop parallel to the cable to maximise the flux through its area.