Chapter 25

Electric Potential

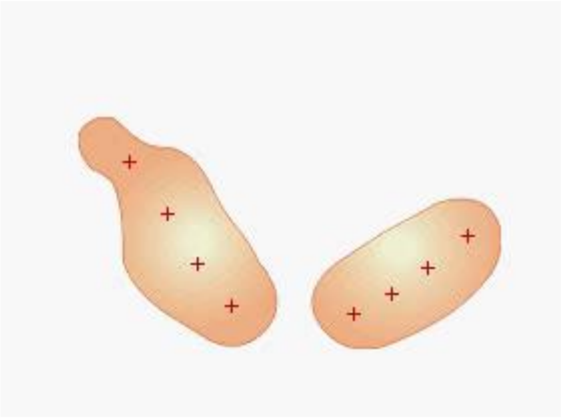
The consultation timetable is now up on the First Year website. Help is available in Room LG02 for 2 hours a day on Mon, Wed, Fri. Check the website for updates.

Weeks 2-13:

Mon	12-2
Wed	1-3
Fri	12-2

Remember:
Quiz 1 is due
Sunday 11pm
18 August 2013

EFM08AN1
Electrostatic Potential Energy



- Lightning captured at over 7000 frames per second

Electrical Potential Energy

- When a test charge is placed in an electric field, it experiences a force
  - $\mathbf{F} = q_o\mathbf{E}$
- The force is conservative
- Now consider the work done moving an infinitesimal displacement, $d\mathbf{s}$, on the path the charge moves
- Work done by electric field is $\mathbf{F} \cdot d\mathbf{s} = q_o\mathbf{E} \cdot d\mathbf{s}$

Electric Potential Energy, cont

- Work done by electric field is $\mathbf{F}\cdot d\mathbf{s} = q_o\mathbf{E}\cdot d\mathbf{s}$
- As this work is done by the field, the potential energy of the charge-field system is changed by $\Delta U = -q_o\mathbf{E}\cdot d\mathbf{s}$
- For a finite displacement of the charge from A to B, the change in potential energy is

$$\Delta U = U_B - U_A = -q_o \int_A^B \mathbf{E} \cdot d\mathbf{s}$$

- Because $q_o\mathbf{E}$ is conservative, the line integral does not depend on the path taken by the charge

Electric Potential

- The potential energy per unit charge, U/q_o, is the **electric potential**
  - The potential is independent of the value of q_o
  - The potential has a value at every point in an electric field
- The electric potential is $V = \dfrac{U}{q_o}$

Electric Potential, cont.

- The potential is a scalar quantity
  - Since energy is a scalar
- As a charged particle moves in an electric field, it will experience a change in potential

$$\Delta V = \frac{\Delta U}{q_o} = -\int_A^B \mathbf{E} \cdot d\mathbf{s}$$

Electric Potential, final

- The *difference* in potential is the meaningful quantity
- We often take the value of the potential to be zero at some convenient point in the field
- Electric potential is a scalar characteristic of an electric field, independent of any charges that may be placed in the field

Work and Electric Potential

- Assume a charge moves in an electric field without any change in its kinetic energy
- The work performed on the charge is

  $W = \Delta U = q\,\Delta V$

Units of Electric Potential

- 1 V = 1 J/C
  - V is a volt
  - It takes one joule of work to move a 1-coulomb charge through a potential difference of 1 volt
- In addition, 1 N/C = 1 V/m
  - This indicates we can interpret the electric field as a measure of the rate of change with position of the electric potential

Electron-Volts

- One ***electron-volt*** is defined as the energy a charge-field system gains or loses when a charge of magnitude *e* (an electron or a proton) is moved through a potential difference of 1 volt
  - $1 \text{ eV} = 1.60 \times 10^{-19} \text{ J}$
- Commonly used in atomic and nuclear physics

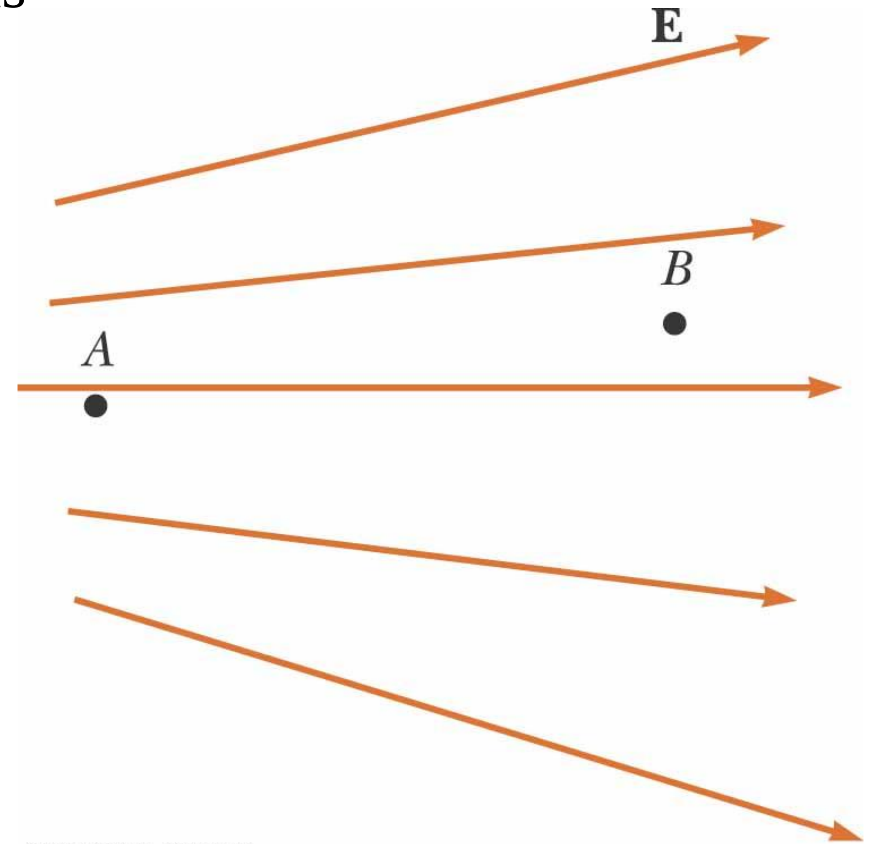
Quick Quiz 25.2

In this figure, a negative charge is placed at *A* and then moved to *B*. The change in potential energy of the charge–field system for this process is

(a) positive

(b) negative

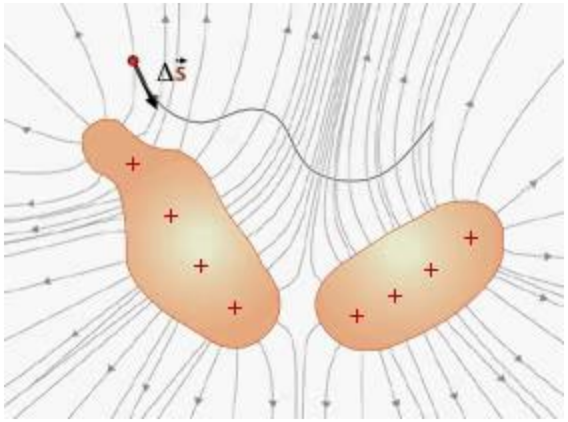
(c) zero



Quick Quiz 25.2

Answer: (a). The potential energy of the system changes by $\Delta U = -q_o\mathbf{E}\cdot d\mathbf{s}$. So, if a negative test charge is moved along the direction of the field lines, the change in potential energy is positive. Work must be done to move the charge in the direction opposite to the electric force on it.

EFM08AN2
Electric Potential Difference



Potential Difference in a Uniform Field

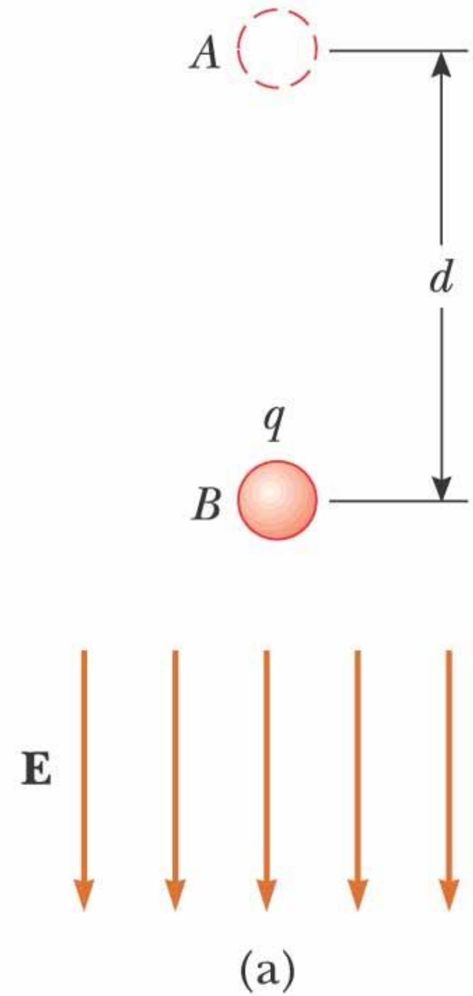
- The equations for electric potential can be simplified if the electric field is uniform:

$$V_B - V_A = \Delta V = -\int_A^B \mathbf{E} \cdot d\mathbf{s} = -E\int_A^B d\mathbf{s} = -Ed$$

- The negative sign indicates that the electric potential at point *B* is lower than at point *A*

Energy and the Direction of Electric Field

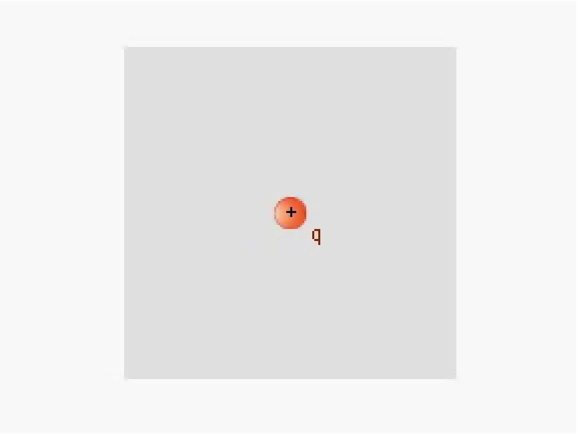
- When the electric field is directed downward, point *B* is at a lower potential than point *A*
- When a positive test charge moves from *A* to *B*, the charge-field system loses potential energy



Conservation of Energy

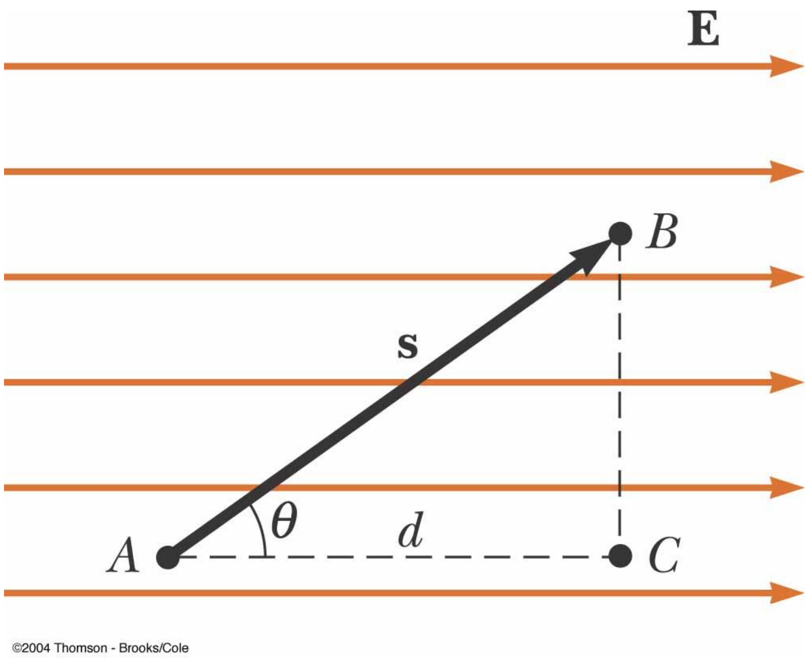
- An electric field does work on a positive charge when the charge moves in the direction of the electric field
- The charged particle gains kinetic energy equal to the potential energy lost
  - Conservation of Energy
- If q_o is negative, then ΔU is positive
  - The system *gains* potential energy when the charge moves in the direction of the field, since an external agent does work on the charge

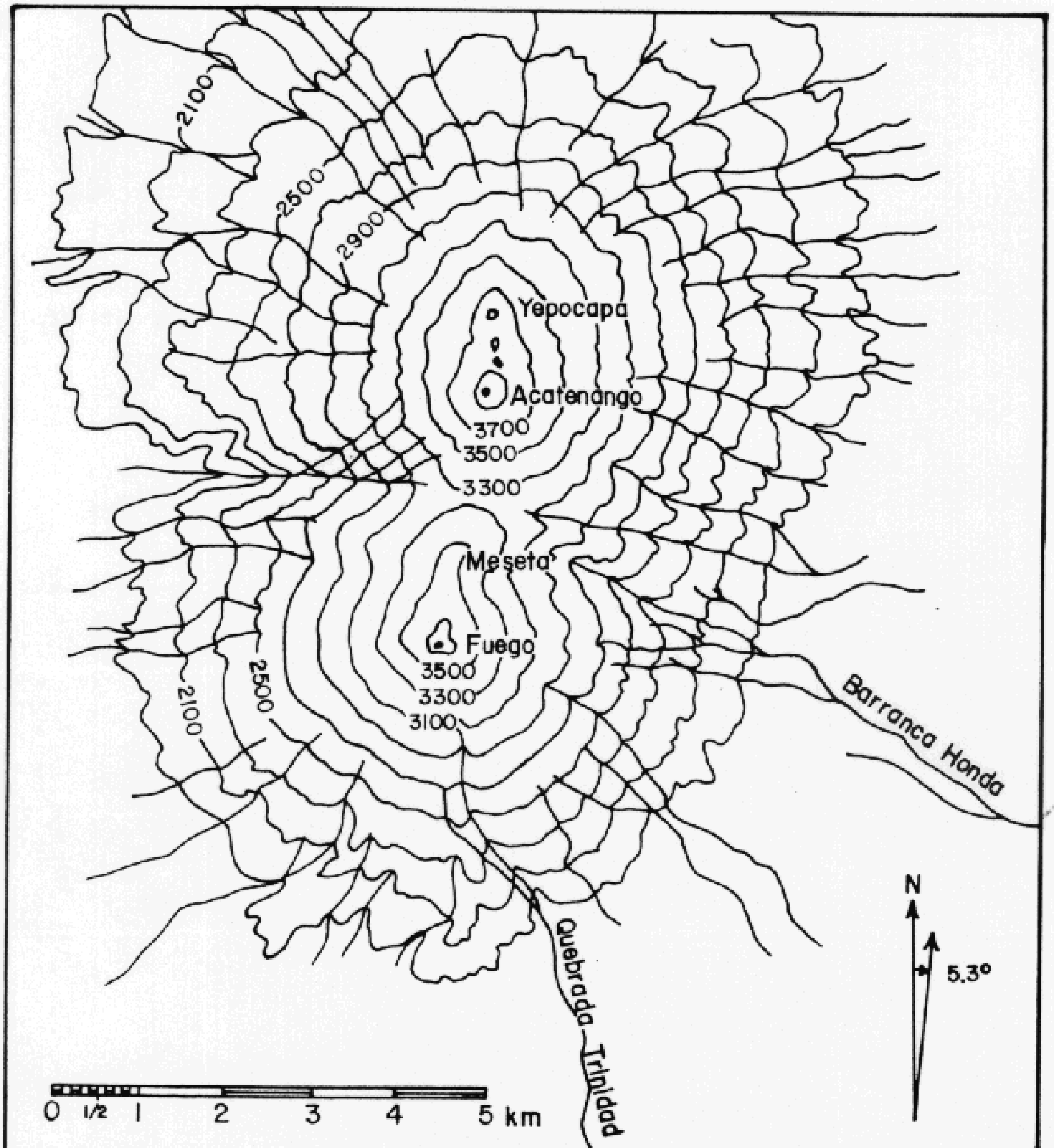
EFA09AN1 Equipotential Surfaces



Equipotential Surface

- Point *B* is at a lower potential than point *A*
- Points *B* and *C* are at the same potential
- The name **equipotential surface** is given to any surface consisting of a continuous distribution of points having the same electric potential





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Quick Quiz 25.4

For the equipotential surfaces in this figure, what is the *approximate* direction of the electric field?

(a) Out of the page

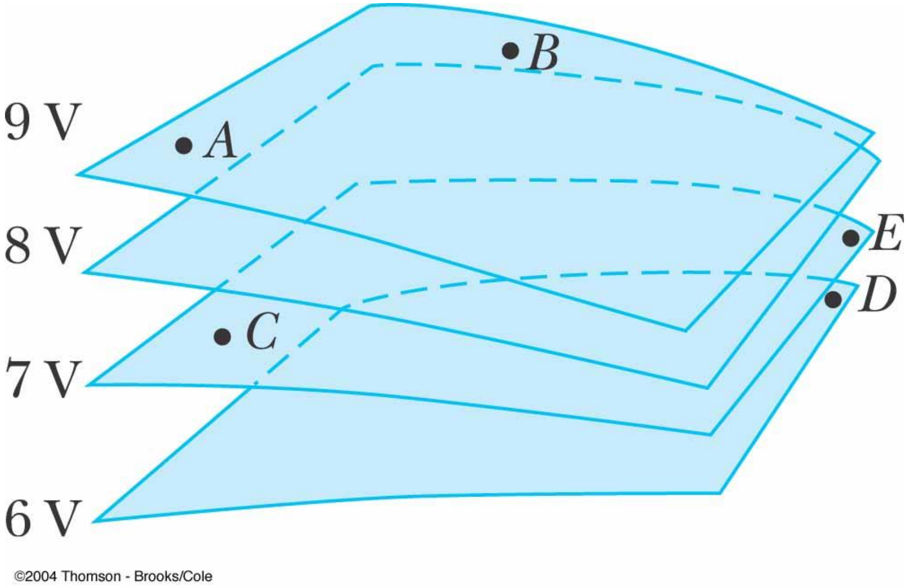
(b) Into the page

(c) Toward the right edge of the page

(d) Toward the left edge of the page

(e) Toward the top of the page

(f) Toward the bottom of the page



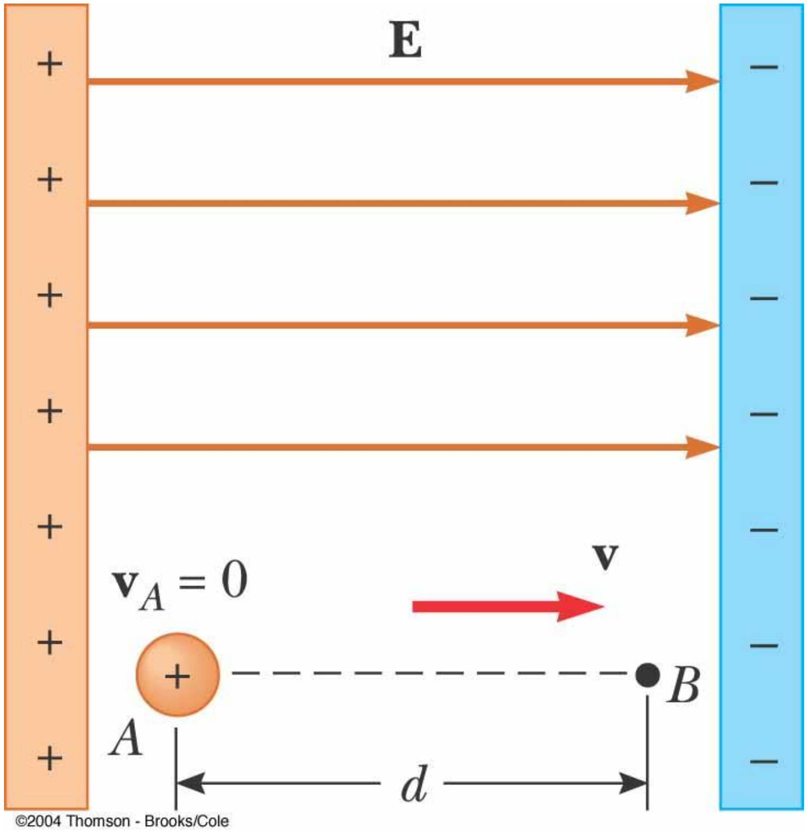
NOTE: by "page" we mean the surface on which this Quiz is displayed, which is perpendicular to the blue sheets in the diagram.

Quick Quiz 25.4

Answer: (f). The electric field points in the direction of decreasing electric potential.

Charged Particle in a Uniform Field, Example

- A positive charge is released from rest and moves in the direction of the electric field
- The change in potential is negative
- The change in potential energy is negative
- The force and acceleration are in the direction of the field
- *Calculate change in electric potential, in PE and speed of proton, moving from A to B*
  - Example 25.2

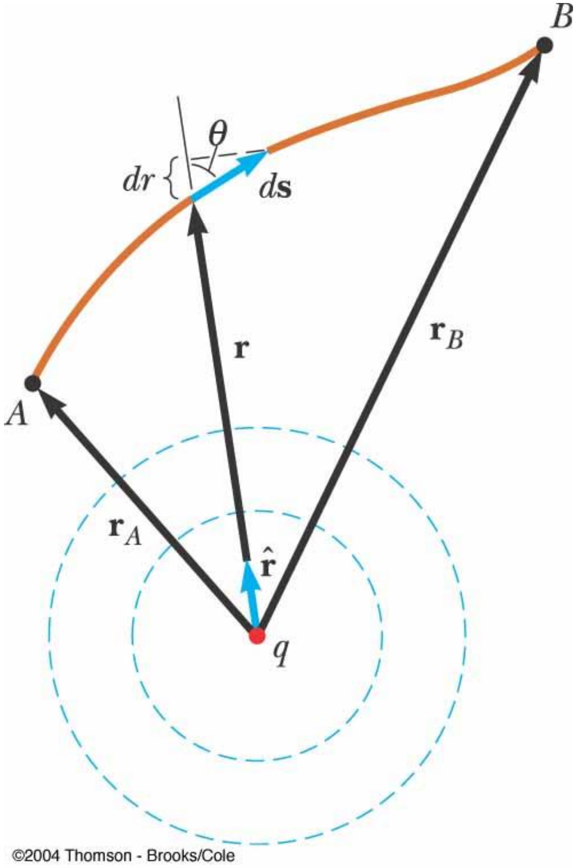


Potential and Point Charges

- A positive point charge produces a field directed radially outward
- The potential difference between points *A* and *B* will be

$$V_B - V_A = -\int_{\mathbf{A}}^{\mathbf{B}} \mathbf{E}\,.d\mathbf{s} = -k_e \int_{r_A}^{r_B} \frac{q}{r^2}\,dr$$

$$= k_e q \left[\frac{1}{r_B} - \frac{1}{r_A} \right]$$

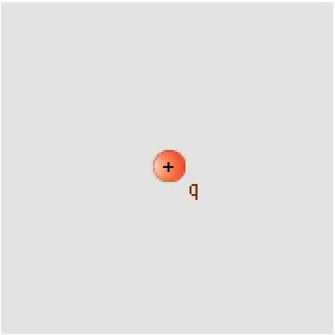


Potential and Point Charges

- The electric potential is independent of the path between points *A* and *B*
- It is customary to choose a reference potential of $V = 0$ at $r_A = \infty$
- Then the potential at some point *r* is

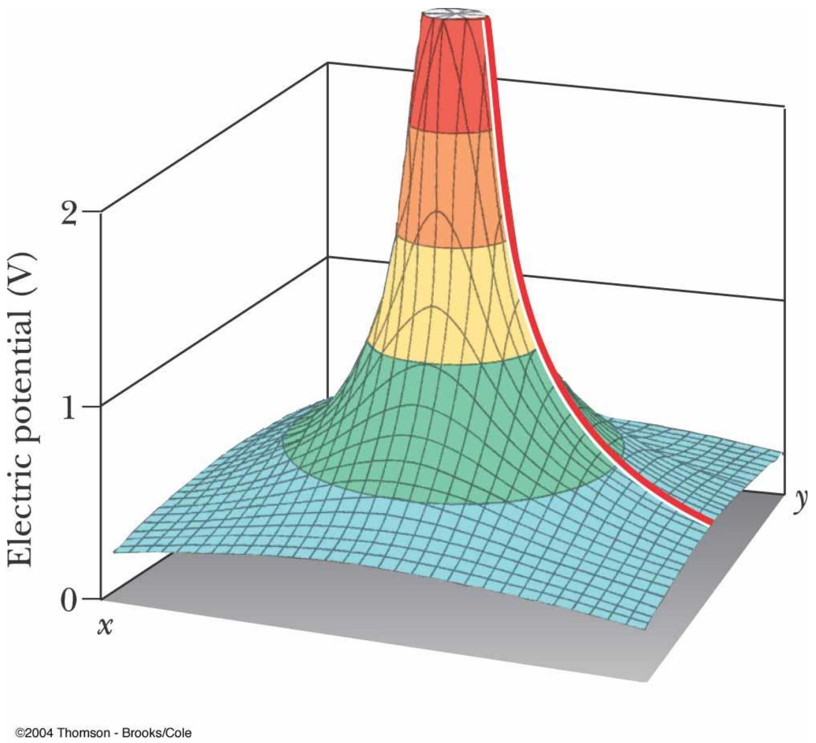
$$V = k_e \frac{q}{r}$$

EFM09AN1
How to find the electric field from the electric potential



Electric Potential of a Point Charge

- The electric potential in the plane around a single point charge is shown
- The red line shows the 1/*r* nature of the potential



Electric Potential with Multiple Charges

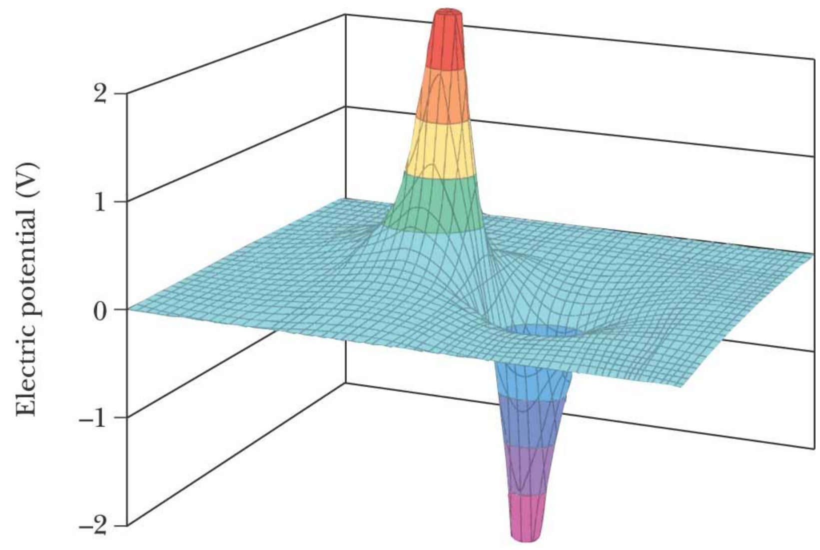
- The electric potential due to several point charges is the sum of the potentials due to each individual charge
  - Example of the superposition principle
  - The sum is the algebraic sum

$$V = k_e \sum_i \frac{q_i}{r_i}$$

  - $V = 0$ at $r = \infty$

Electric Potential of a Dipole

- The graph shows the potential (y-axis) of an electric dipole
- The steep slope between the charges represents the strong electric field in this region

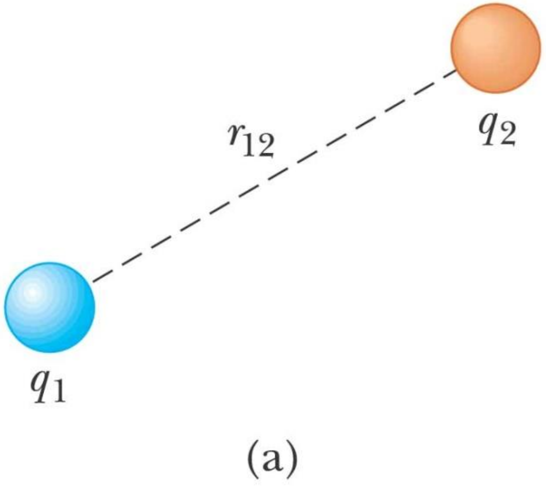


Potential Energy of Multiple Charges

- Consider two charged particles
- The potential energy of the system is

$$U = q_1 V = q_1 \frac{k_e q_2}{r_{12}} = k_e \frac{q_1 q_2}{r_{12}}$$

- If both charges of the same sign then work must be done to bring them together

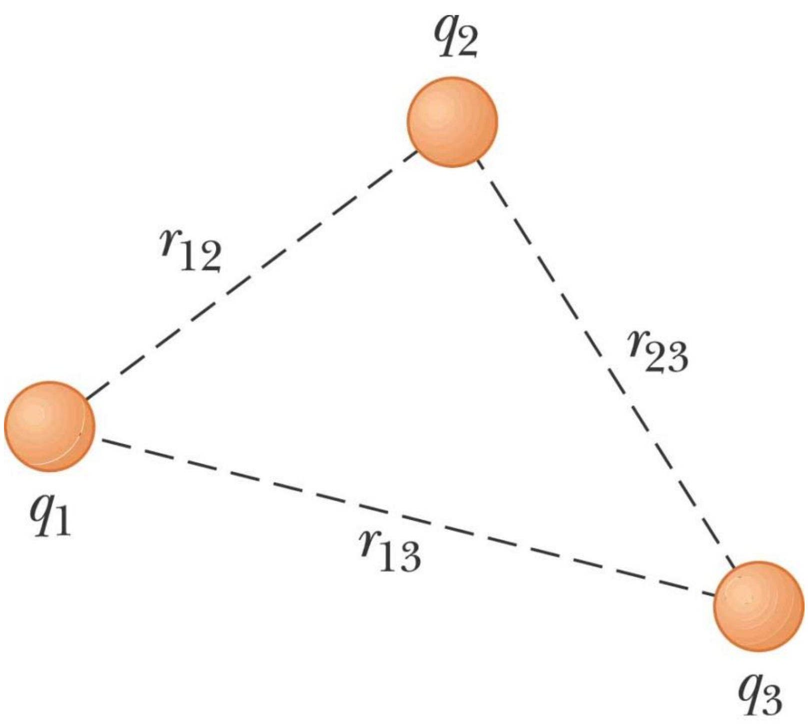


U with Multiple Charges, final

- If there are more than two charges, then find *U* for each pair of charges and add them
- For three charges:

$$U = k_e \left(\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}} \right)$$

  - The result is independent of the order of the charges



Quick Quiz 25.5a

A spherical balloon contains a positively charged object at its centre. As the balloon is inflated to a greater volume while the charged object remains at the centre, the electric potential at the surface of the balloon will

(a) increase

(b) decrease

(c) remain the same.

Quick Quiz 25.5a

Answer: (b). The electric potential is inversely proportional to the radius $\left(V = k_e \dfrac{q}{r}\right)$

Quick Quiz 25.5b

Recall that the spherical balloon from part a) contains a positively charged object at its centre. As the balloon is inflated to a greater volume while the charged object remains at the centre, the electric flux through the surface of the balloon will

(a) increase

(b) decrease

(c) remain the same.

Quick Quiz 25.5b

Answer: (c). Because the same number of field lines passes through a closed surface of any shape or size, the electric flux through the surface remains constant.

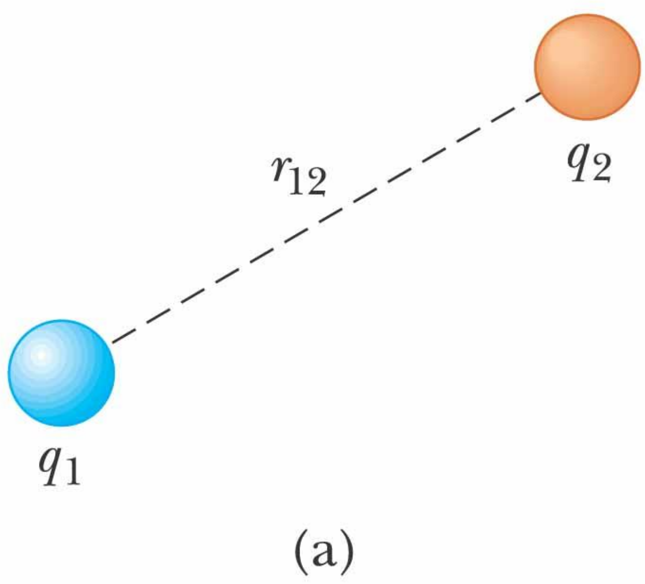
Quick Quiz 25.6

In the figure, take q_1 to be a negative source charge and q_2 to be the test charge. If q_2 is initially positive and is changed to a charge of the same magnitude but negative, the potential at the position of q_2 due to q_1

(a) increases

(b) decreases

(c) remains the same



Quick Quiz 25.6

Answer: (c). The potential is established only by the source charge and is independent of the test charge.

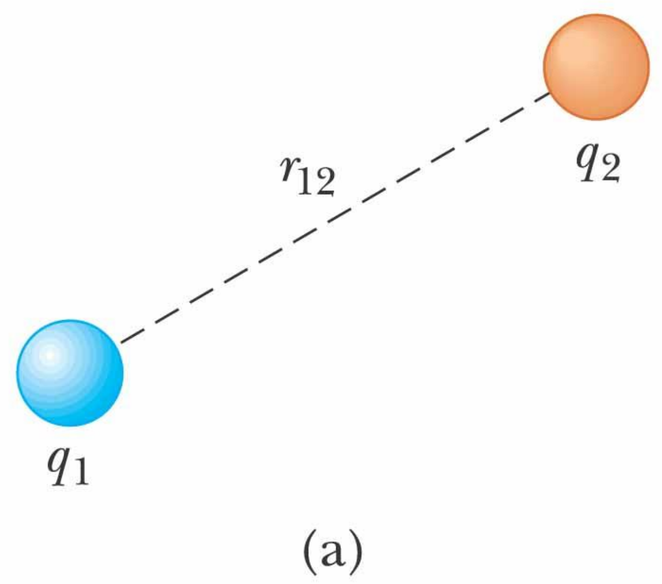
Quick Quiz 25.7

Consider the situation in question 25.6 again. When q_2 is changed from positive to negative, the potential energy of the two-charge system

(a) increases

(b) decreases

(c) remains the same



Quick Quiz 25.7

Answer: (a). The potential energy of the two-charge system is initially negative, due to the products of charges of opposite sign in $U = k_e\, q_1 q_2 / r_{12}$. When the sign of q_2 is changed, both charges are negative, and the potential energy of the system is positive.

Finding **E** From *V*

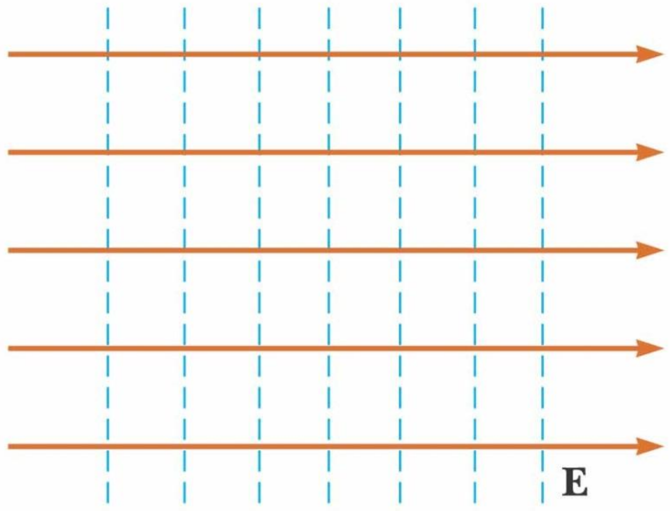
- Assume, to start, that **E** has only an *x* component. Then from $\Delta V = -\boldsymbol{E}.\boldsymbol{ds} = -E_x dx$

$$E_x = -\frac{dV}{dx}$$

- Similar statements apply to the *y* & *z* components
- Along an equipotential surfaces $\Delta V = 0$
  - Hence $\boldsymbol{E} \perp \boldsymbol{ds}$
  - i.e. an equipotential surface is perpendicular to the electric field lines passing through it

E and *V* for an Infinite Sheet of Charge

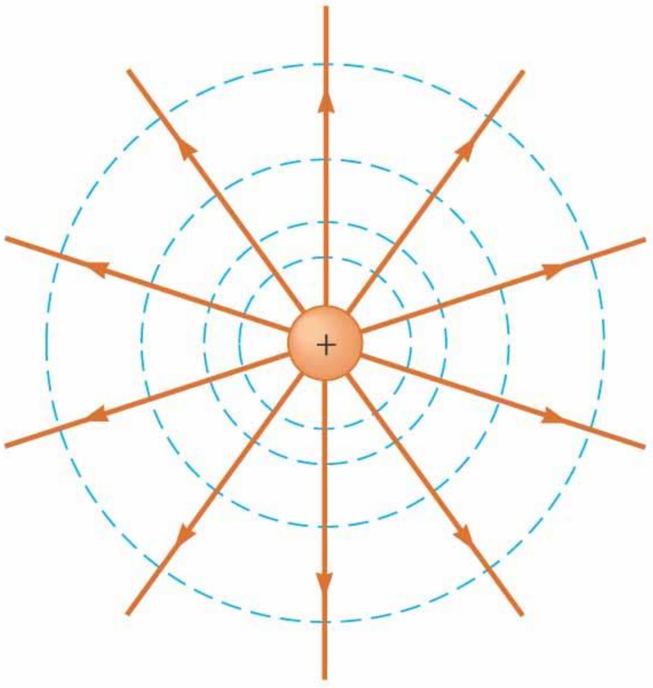
- The equipotential lines are the dashed blue lines
- The electric field lines are the brown lines
- The equipotential lines are everywhere perpendicular to the field lines



(a)

E and *V* for a Point Charge

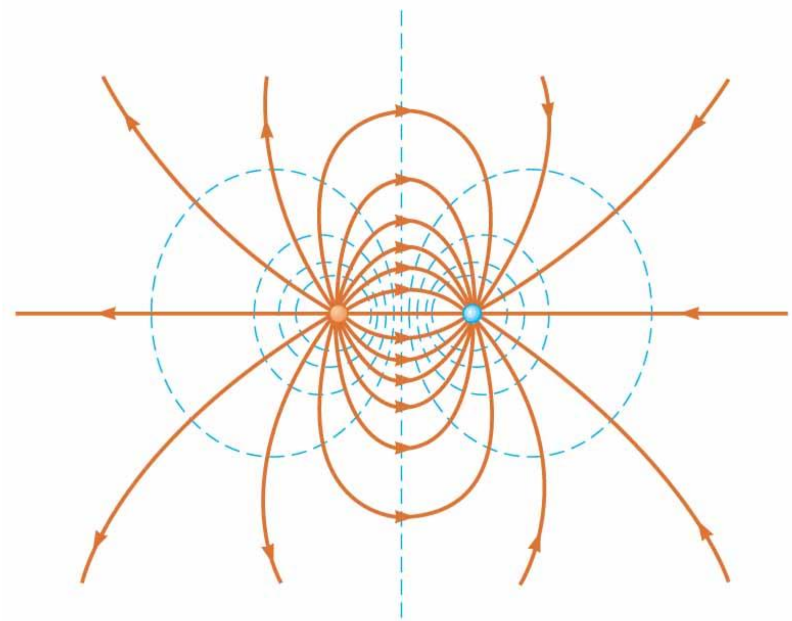
- The equipotential lines are the dashed blue lines
- The electric field lines are the brown lines
- The equipotential lines are everywhere perpendicular to the field lines



(b)

E and *V* for a Dipole

- The equipotential lines are the dashed blue lines
- The electric field lines are the brown lines
- The equipotential lines are everywhere perpendicular to the field lines



(c)

Electric Field from Potential, General

- In general, the electric potential is a function of all three dimensions
- Given $V(x, y, z)$ you can find E_x, E_y and E_z as partial derivatives

$$E_x = -\frac{\partial V}{\partial x} \quad E_y = -\frac{\partial V}{\partial y} \quad E_z = -\frac{\partial V}{\partial z}$$

Quick Quiz 25.8

In a certain region of space, the electric potential is zero everywhere along the *x* axis. From this we can conclude that the *x* component of the electric field in this region is

(a) zero

(b) in the *x* direction

(c) in the –*x* direction.

Quick Quiz 25.8

Answer: (a). If the potential is constant (zero in this case), its derivative along this direction is zero.

Quick Quiz 25.9

In a certain region of space, the electric field is zero. From this we can conclude that the electric potential in this region is

(a) zero

(b) constant

(c) positive

(d) negative

Quick Quiz 25.9

Answer: (b). If the electric field is zero, there is no change in the electric potential and it must be constant. This constant value *could be* zero but does not *have to be* zero.

Potentials Due to Various Charge Distributions

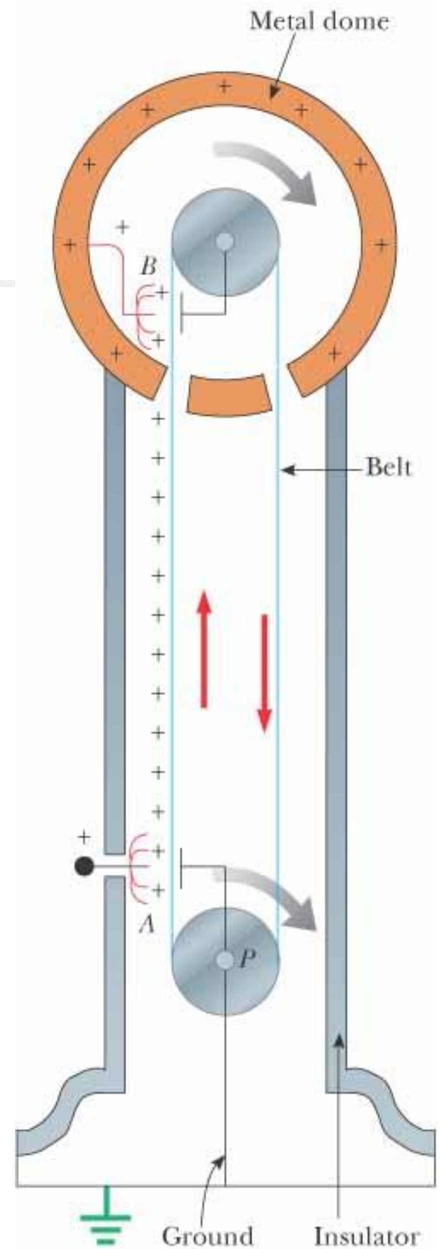
Table 25.1

Electric Potential Due to Various Charge Distributions

Charge Distribution	Electric Potential	Location
Uniformly charged ring of radius a	$V = k_e \frac{Q}{\sqrt{x^2 + a^2}}$	Along perpendicular central axis of ring, distance x from ring center
Uniformly charged disk of radius a	$V = 2\pi k_e \sigma[(x^2 + a^2)^{1/2} - x]$	Along perpendicular central axis of disk, distance x from disk center
Uniformly charged, *insulating* solid sphere of radius R and total charge Q	$V = k_e \frac{Q}{r}$	$r \geq R$
	$V = \frac{k_e Q}{2R}\left(3 - \frac{r^2}{R^2}\right)$	$r < R$
Isolated *conducting* sphere of radius R and total charge Q	$V = k_e \frac{Q}{r}$	$r > R$
	$V = k_e \frac{Q}{R}$	$r \leq R$

Van de Graaff Generator

- Charge is delivered continuously to a high-potential electrode by means of a moving belt of insulating material
- The high-voltage electrode is a hollow metal dome mounted on an insulated column
  - Treat as a spherical conductor:
    - $E=kQ/r^2$, $V=kQ/r$
- Large potentials can be developed by repeated trips of the belt
  - Protons accelerated through such large potentials receive enough energy to initiate nuclear reactions



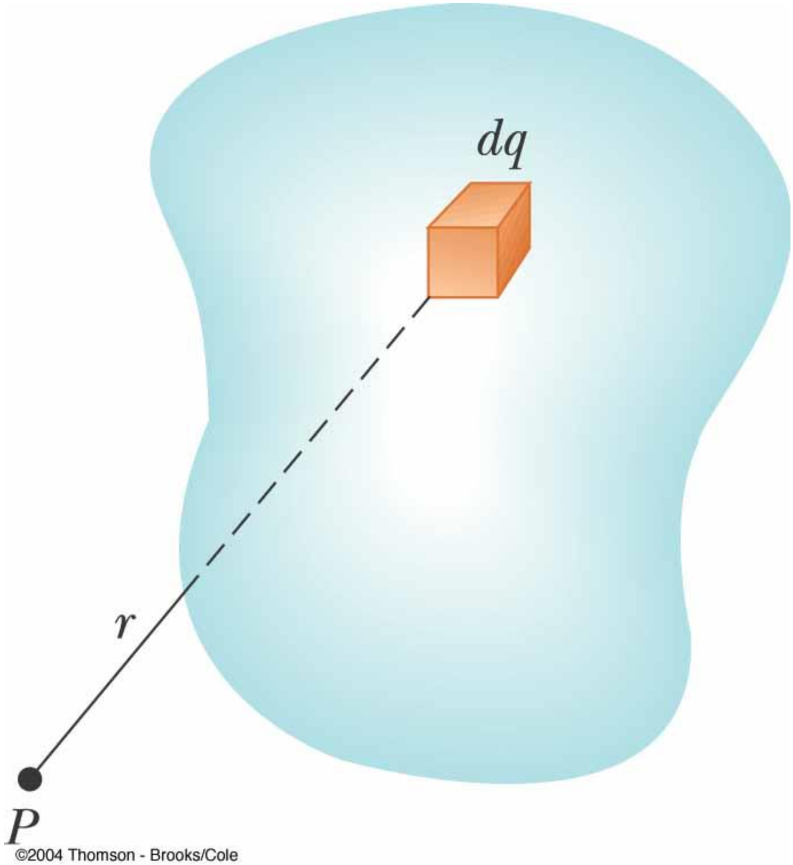
Demo EA4: Charged Particles and Electrostatic Pinwheel

- Pinwheel rotates away from the points as the corona discharge gives the air a space charge which is of the same sign as the point.

QuickTime™ and a
TIFF (Uncompressed) decompressor
are needed to see this picture.

Electric Potential for a Continuous Charge Distribution

- Consider a small charge element *dq*
  - Treat as point charge
- The potential at some point due to this charge element is $dV = k_e \frac{dq}{r}$



V for a Continuous Charge Distribution, cont.

- To find the total potential, you need to integrate to include the contributions from all the elements

$$V = k_e \int \frac{dq}{r}$$

  - This value for *V* uses the reference of *V* = 0 when *P* is infinitely far away from the charge distributions

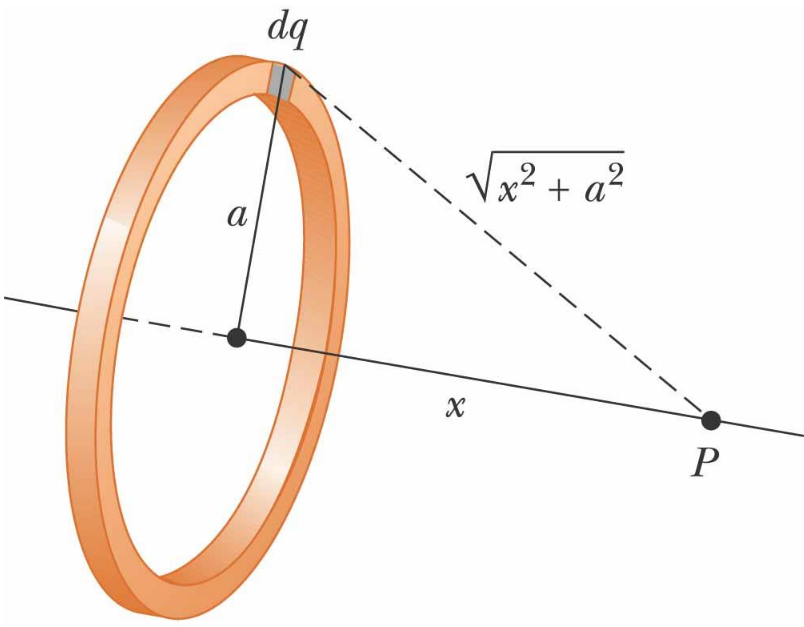
V for Uniformly Charged Ring

- *P* is located on the perpendicular central axis of the uniformly charged ring
  - The ring has a radius *a* and a total charge *Q*

$$V = k_e \int \frac{dq}{r} = \frac{k_e Q}{\sqrt{x^2 + a^2}}$$

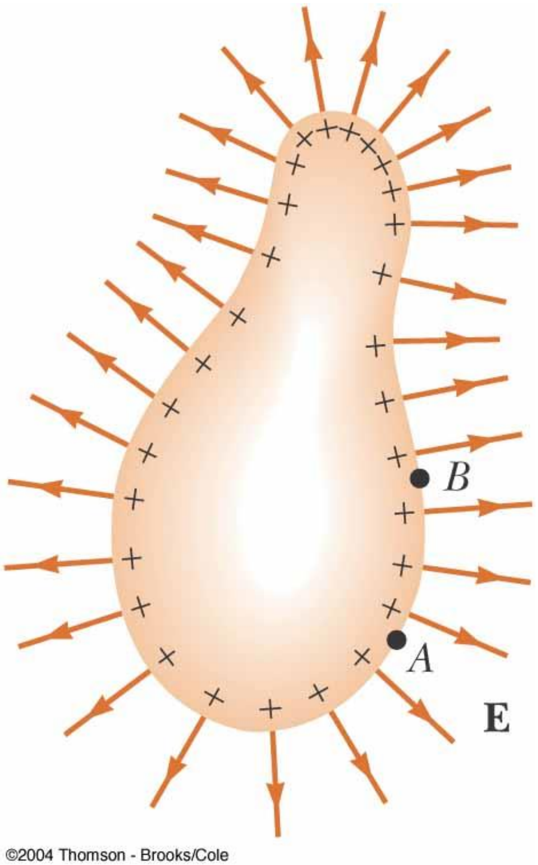
- *See example 25.5 for derivation*

  [Do it yourself first!]



V Due to a Charged Conductor

- Consider two points on the surface of the charged conductor as shown
- **E** is always perpendicular to the displacement *d***s**
- Therefore, **E** · *d***s** = 0
- Therefore, the potential difference between *A* and *B* is also zero

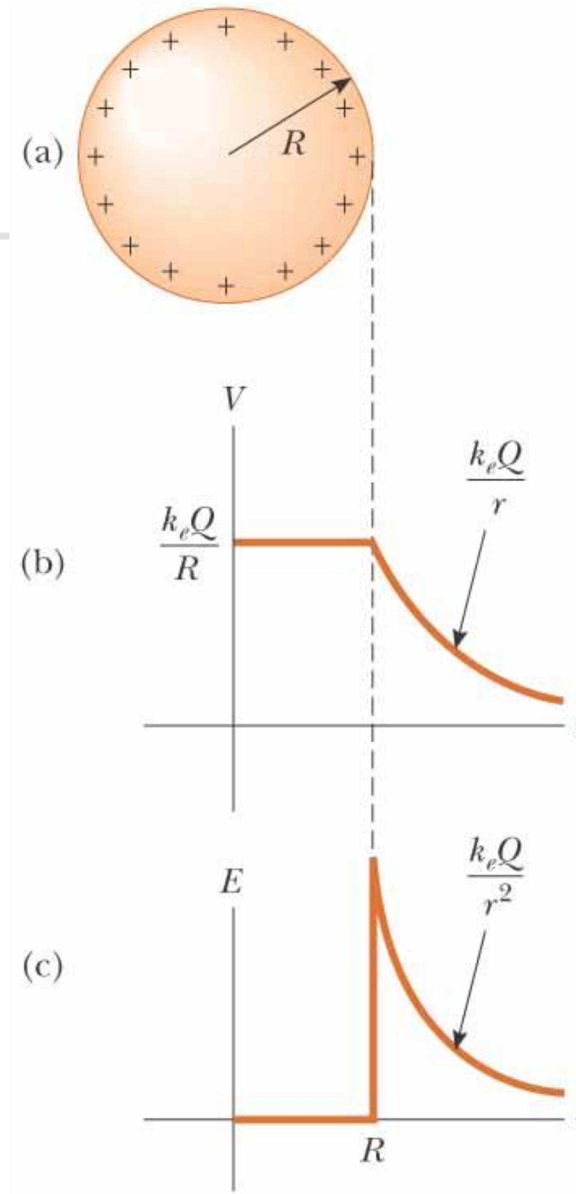


V Due to a Charged Conductor, cont.

- *V* is constant everywhere on the surface of a charged conductor in equilibrium
  - $\Delta V = 0$ between any two points on the surface
- The surface of any charged conductor in electrostatic equilibrium is an equipotential surface
- Because the electric field is zero inside the conductor, we conclude that the electric potential is constant everywhere inside the conductor and equal to the value at the surface

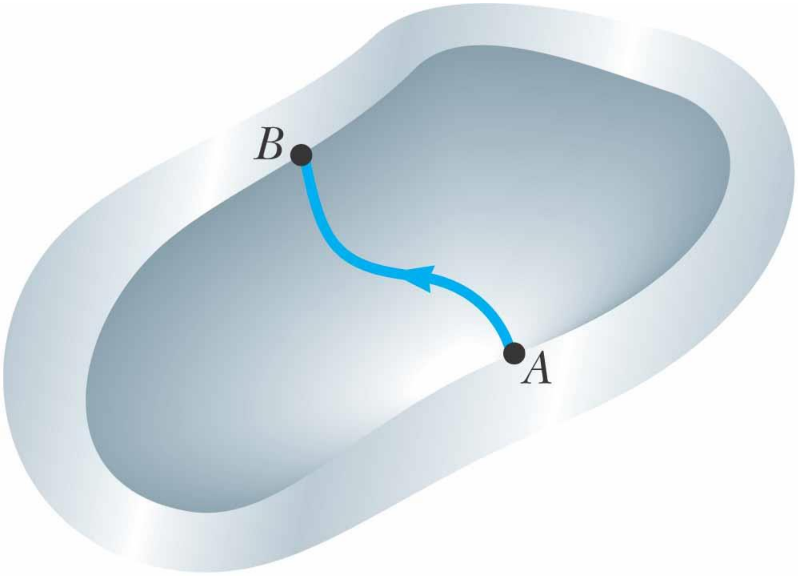
E Compared to V

- The electric potential outside a spherically symmetric charge distribution drops linearly with $1/r$
- The electric field drops as $1/r^2$
- The effect of a charge on the space surrounding it:
  - The charge sets up a vector electric field which is related to the force
  - The charge sets up a scalar potential which is related to the energy



Cavity in a Conductor

- Assume an irregularly shaped cavity is inside a conductor
- Assume no charges are inside the cavity
- The electric field inside the conductor must be zero (proof on next slide)



Cavity in a Conductor: Proof

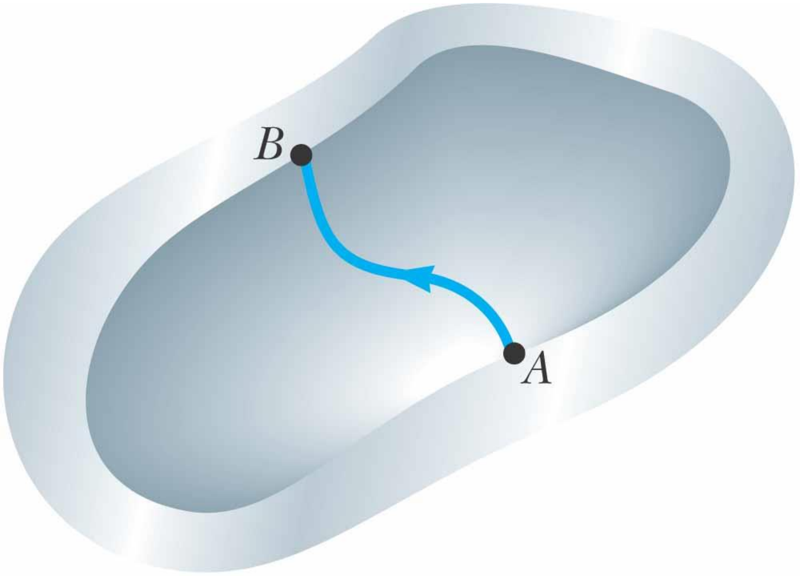
- Every point on the conductor at the same electric potential.
- So, for all paths between *A* and *B*,

$$V_B - V_A = -\int_A^B \mathbf{E} \cdot d\mathbf{s} = 0$$

- Thus ***E*** must be zero inside conductor.
  - i.e. a cavity surrounded by conducting walls is a field-free region as long as no charges are inside the cavity.

Cavity in a Conductor: application

- *Application in protecting electronic circuits. Stray electric fields will cause charges to re-distribute on surface of box, and so cancel any strong fields inside. No electric field can penetrate inside.*



End of Chapter

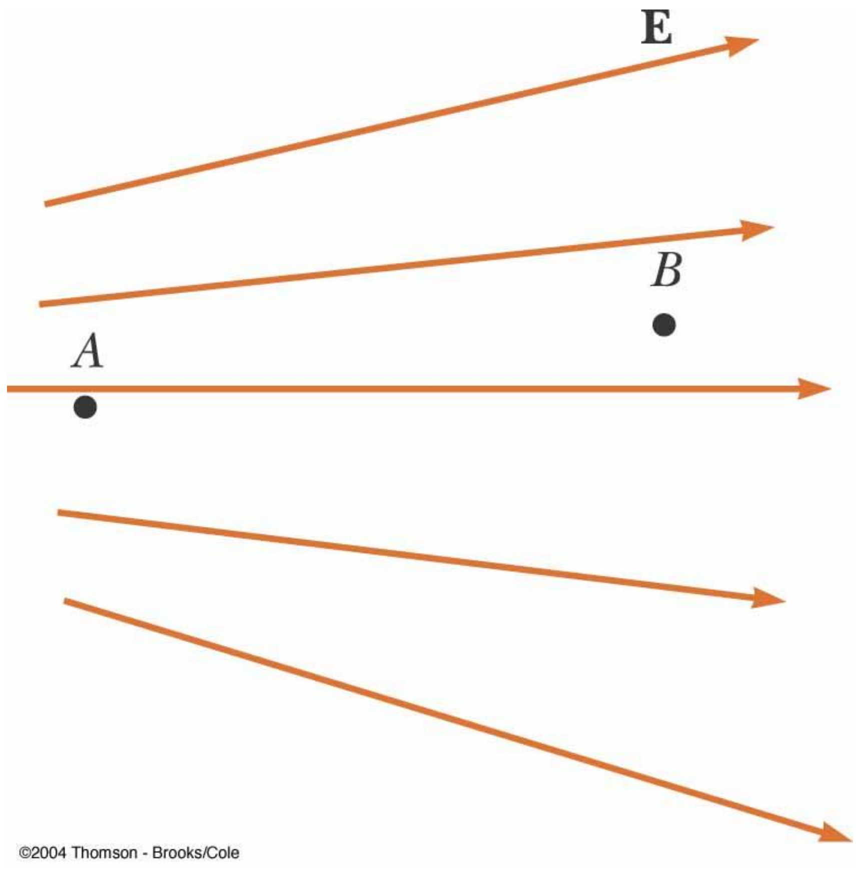
Quick Quiz 25.1

In the figure below, two points A and B are located within a region in which there is an electric field. The potential difference $\Delta V = V_B - V_A$ is

(a) positive

(b) negative

(c) zero



Quick Quiz 25.1

Answer: (b). When moving straight from A to B, $\mathbf{E}$ and $d\mathbf{s}$ both point toward the right. Thus, the dot product $\mathbf{E} \cdot d\mathbf{s}$ is positive and ΔV is negative.

Quick Quiz 25.3

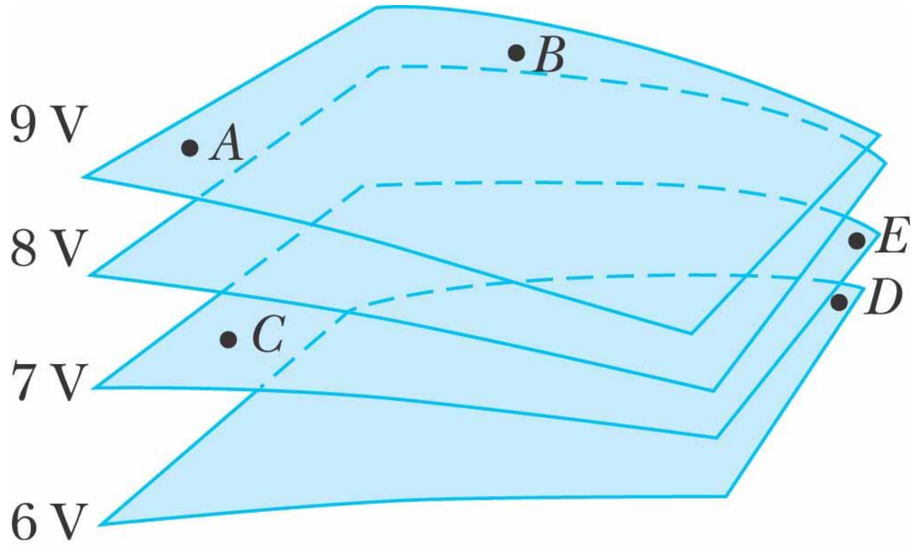
The labeled points of the figure below are on a series of equipotential surfaces associated with an electric field. Rank (from greatest to least) the work done by the electric field on a positively charged particle that moves along the following transitions.

(a) *A* -> *B*, *B* -> *C*, *C* -> *D*, *D* -> *E*

(b) *A* -> *B*, *D* -> *E*, *B* -> *C*, *C* -> *D*

(c) *B* -> *C*, *C* -> *D*, *A* -> *B*, *D* -> *E*

(d) *D* -> *E*, *C* -> *D*, *B* -> *C*, *A* -> *B*

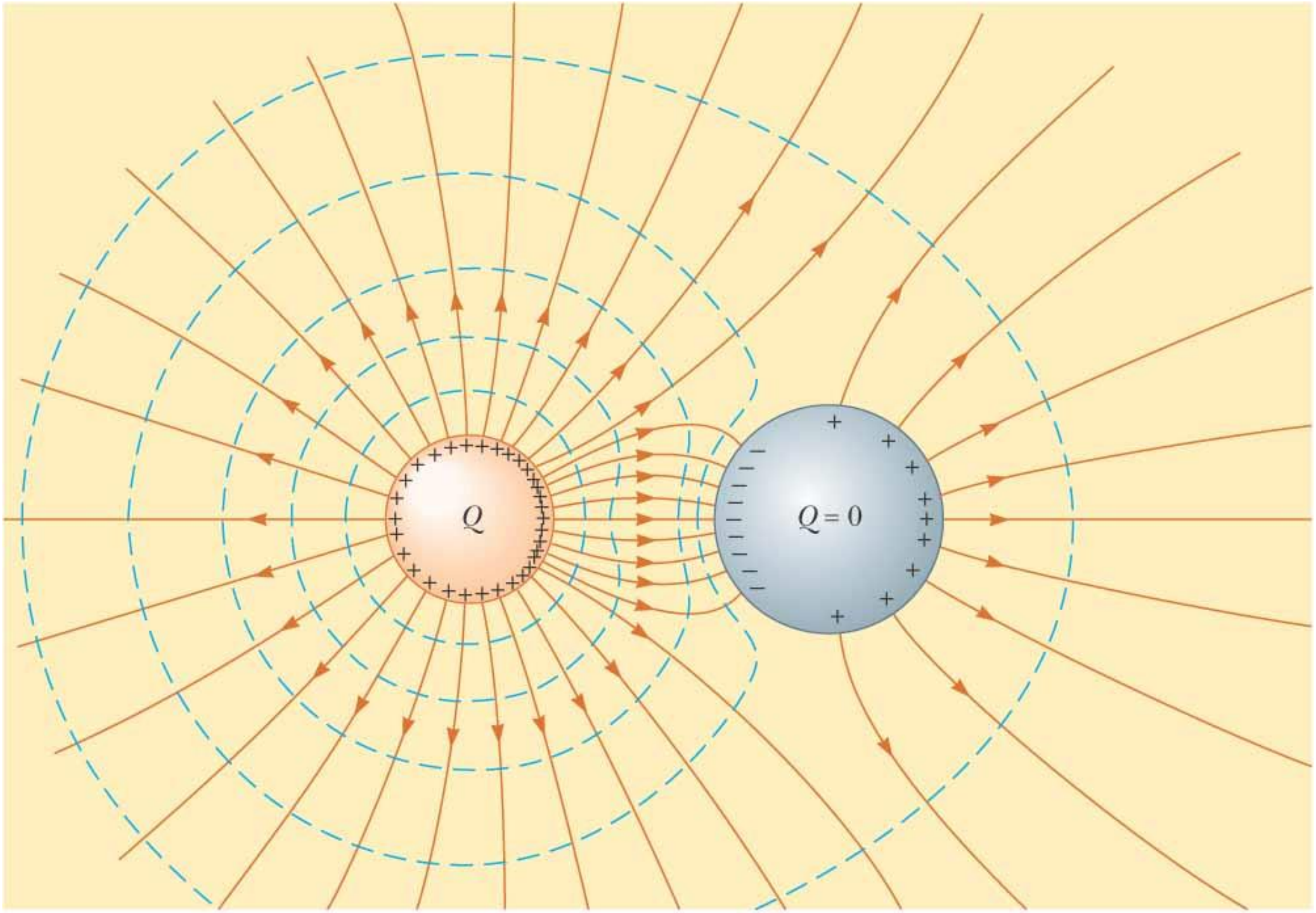


Quick Quiz 25.3

Answer: (c). Moving from *B* to *C* decreases the electric potential by 2 V, so the electric field performs 2 J of work on each coulomb of positive charge that moves. Moving from *C* to *D* decreases the electric potential by 1 V, so 1 J of work is done by the field. It takes no work to move the charge from *A* to *B* because the electric potential does not change. Moving from *D* to *E* increases the electric potential by 1 V, and thus the field does –1 J of work per unit of positive charge that moves.

Quick Quiz 25.10

Consider starting at the centre of the left-hand sphere (sphere 1, of radius *a*) in the figure below and moving to the far right of the diagram, passing through the centre of the right-hand sphere (sphere 2, of radius *c*) along the way. The centres of the spheres are a distance *b* apart. Draw a graph of the electric potential as a function of position relative to the centre of the left-hand sphere.



Quick Quiz 25.10

Answer: The graph would look like the sketch below. Notice the flat plateaus at each conductor, representing the constant electric potential inside a solid conductor.